Risk-Return Tradeoff in the Pacific Basin Equity Markets

Ai-Ru Cheng* Mohammad R. Jahan-Parvar†
University of California, Santa Cruz East Carolina University ‡

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Abstract

We conduct an empirical study of risk-return tradeoff in fourteen Pacific basin equity markets using several volatility estimators, including three variants of GARCH class, equally weighted rolling window volatility, and mixed data sampling (MIDAS), as well as binormal GARCH (BiN-GARCH) model which allows for non-zero conditional skewness in returns. Our findings imply that the BiN-GARCH model, which allows for time-variation in the conditional skewness and market price of risk, captures the expected positive risk-return relationship for more than half of the markets studied. In comparison, symmetric skewness models such as MIDAS or GARCH variants fail to capture positive and statistically significant market price of risk estimates. These results provide support for the growing literature on the necessity of modeling conditional higher moments in financial research.

Keywords: Conditional variance, GARCH, Intertemporal CAPM, Mixed data sampling, Pacific basin equity markets, Risk-return trade-off.
JEL classification: C22; G12; G15.

*Corresponding Author, Assistant Professor, Department of Economics, University of California, Santa Cruz, CA 95064, USA, Phone No: (831) 459-2318, e-mail: archeng@ucsc.edu.
†Assistant Professor, Department of Economics, East Carolina University, Brewster A-426, Greenville, NC 27858-4353, USA, Phone No: (252) 328-4770, e-mail: jahanparvarm@ecu.edu.
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1 Introduction

In the past twenty years, many studies in empirical asset pricing have devoted considerable energy in testing the systematic trade-off between expected returns and risk, characterized by Merton (1973) intertemporal capital asset pricing (ICAPM). The majority of empirical studies postulate a positive and linear relation between the conditional variance and expected excess market returns, with time-invariant market price of risk. Moreover, the majority of studies assume symmetry in the conditional distribution of excess returns. These assumptions essentially translate into a race to come up with the best model for the conditional volatility.

However, there is no consensus on even the most basic theoretical properties of the risk-return relationship; see Rossi and Timmermann (2009). In this study, we show that the problem is not the volatility model used, but failure to account for conditional higher moments and their role in the risk-return relationship. We use several variants of existing empirical methodologies to test the risk-return relationship in 14 Pacific rim financial markets. We find that the model that does not require symmetry in the conditional distribution of excess returns outperforms those that require this restriction.

We find empirical supporting evidence in favor of a positive risk-return relationship in eight markets studied, using binormal GARCH (henceforth, BiN-GARCH) methodology of Feunou et al. (2010). Other methods, including rolling estimator of French et al. (1987), various GARCH specifications, or mixed data sampling (henceforth, MIDAS) of Ghysels et al. (2005, 2006, 2007), deliver significantly weaker results.

We believe that our results provide empirical support for the idea that to build reliable risk management measures in the Pacific basin markets, one needs to consider modeling conditional higher moments such as conditional skewness, in addition to the traditional modeling of the first two conditional moments. This paper contributes to the existing literature in two important directions. First, it provides an empirical assessment for the ability of several traditional and more recent econometric methodologies in gauging the elusive risk-return trade-off relationship, in the context of non-U.S. markets.

Second, this paper empirically demonstrates the impact of asymmetry in conditional distribution of returns, non-linearity in the risk-return trade-off relation, and time-variation in the conditional skewness across the most important region in the world economy in the 21st century.

In general, empirical evidence on risk-return trade-off is quite inconclusive. Campbell (1987), Nelson (1991), and more recently Brandt and Kang (2004), find a significantly negative conditional relationship. Harvey (1989) and Glosten et al. (1993) find both a positive and a negative relation depending on the method used. On the other hand, French et al. (1987), Baillie and DeGennaro (1990), and Campbell and Hentschel (1992) find a positive but mostly insignificant relation between the conditional variance and the conditional expected returns. Ghysels et al. (2005) and Ludvigson
and Ng (2007) find a positive and significant relationship in the U.S. data. In particular, Ghysels et al. (2005) report success in capturing a time-invariant, positive, and statistically significant market price of risk for monthly market returns data from 1928 to 2000. They use daily data for the same period and mixed data sampling (MIDAS) regression methodology to perform statistical assessment of ICAPM. More recently, Rossi and Timmermann (2009) use regression trees to show that the risk-return relationship is state dependent and nonlinear. Feunou et al. (2010) show that the risk-return relationship is nonlinear, time-varying, and crucially depends on the dynamics of conditional skewness of returns. They show that inclusion of conditional skewness in estimation delivers robust, positive risk-return tradeoff in S&P500 and international data.

The rest of the paper is organized as follows. In Section 2, we introduce the data used in this paper. Section 3 introduces the various methodologies used in the paper and discusses the empirical results. Section 4 concludes.

2 Data

We construct market index returns by taking the first difference of logarithm of MSCI (formerly known as Morgan Stanley Capital International) country specific and U.S. dollar denominated indices downloaded from Thomson Reuters’ Datastream. In order to obtain excess returns, we follow the common practice in the finance literature and subtract the three month US Treasury Bill returns from the country specific index returns. T-Bill rates act as the proxy for the risk free rate. Thus, we are analyzing data from the point of view of an international investor that has access to the international fixed income market.

We collected data based on the availability and length of data sets maintained by Datastream. In order to maintain uniformity of results, we use U.S. dollar denominated returns for all the markets. Our sample includes 14 markets from the Pacific rim. Canada and Japan represent the G7 markets. Hong Kong and Singapore are small, rich city-states with highly developed financial markets. Two other “Asian Tigers”, Korea and Taiwan, are also in the sample. Indonesia, Malaysia, and the Philippines are three emerging markets which complete our Asian representatives. An OECD member, Mexico, and Colombia represent Latin America. Australia and New Zealand round up our sample.

As is seen in Table 1, there is wide variation in the behavior of these markets. Most markets have annualized returns in the neighborhood of 2-3% a year. Five markets, Korea, New Zealand, the Philippines, Taiwan, and Thailand have negative excess returns for the sample period of 1988-2009. Reported standard deviations are comparable with what is reported in other empirical studies. All but three markets, Indonesia, Japan, and Korea, demonstrate significant negative skewness. Indonesia and Korea excess returns have positive skewness. Japanese data, on the other hand, do not demonstrate significant departure from unconditional symmetry in returns. All
markets in sample except Colombia seem to have significant excess kurtosis, thus they demonstrate leptokurtotic behavior. We conclude that none of these excess returns series are unconditionally normally distributed.\footnote{We conducted the Jarque-Bera normality test on these series. The null hypothesis of normality is rejected at 1\% or better confidence level for all series. For the sake of brevity, these results are not reported.}

3 Intertemporal Capital Asset Pricing Model

Following Merton (1973),

\[ E_{t-1}[r_t] = \mu + \gamma Var_{t-1}[r_t], \]

where the value of \( \mu \) is expected to be equal to zero and \( \gamma \) is the time-invariant market price of risk. The size and sign of \( \gamma \) dictate the size and direction of risk-return trade-off.\footnote{In Merton’s formulation, \( \gamma \) is the coefficient of relative risk aversion for the representative agent.} Traditionally, we measure the returns as log differences in total return indices. Unfortunately, measuring volatility is considerably more complicated, since volatility is not observable. A vast literature in finance and econometrics is devoted to measurement and assessment of volatility.

To a large extent, the empirical risk-return trade off literature is an exercise finding the “right” volatility measure for \( Var_{t-1}[r_t] \) term in equation (1). Over years, empirical finance community has studied numerous parametric, non-parametric, and hybrid volatility measures. In this paper, we study a relatively large sample of these volatility measures. They include representatives of parametric, non-parametric, and hybrid classes. We find that while this choice of measure is important in capturing the positive market price of risk, it is not sufficient. A successful model of market price of risk seems to require modeling conditional mean, volatility, and skewness of returns. The empirical evidence supporting this claim follows.

3.1 Zero Conditional Skewness in Returns

3.1.1 GARCH Models

We study the risk-return trade-off dynamics in the context of three GARCH-in-Mean models. All the models studied in the section share a common underlying assumption. All these models assume that conditional distribution of returns are symmetric. They may allow for asymmetry in the conditional volatility of the returns, but they maintain that returns themselves are conditionally symmetrically distributed.

The workhorse of this literature remains Bollerslev (1986) GARCH model. In this study, we first posit that the conditional volatility of excess returns follows a simple GARCH(1,1) dynamics.
That is, we assume that the excess returns follow

\[ r_t = \mu + \lambda h_{t-1} + \epsilon_t, \] (2)

where \( \epsilon_t = \sqrt{h_{t-1}} z_t \) and \( z_t \) is a white noise error term. In Bollerslev (1986) formulation, \( h_t \) follows

\[ h_t = \omega + \alpha \epsilon_t^2 + \beta h_{t-1}. \] (3)

These dynamics are stationary when \( \alpha + \beta \leq 1 \). We refer to the combination of equations (2) and (3) as GARCH-in-Mean.

Ghysels et al. (2005) analyze the U.S. “long data” (1928-2000) using the absolute GARCH (Abs-GARCH) specification for robustness purposes. This model specifies the dynamics of the conditional volatility as

\[ h_{1/2}^t = \omega + \alpha |\epsilon_t| + \beta h_{1/2}^{t-1}. \] (4)

We refer to the combination of equations (2) and (4) as Abs-GARCH-in-Mean.

Nelson (1991) exponential GARCH (EGARCH) captures “the leverage effect”. Very concisely, leverage effect is a negative correlation between past returns and future volatility. This model allows for asymmetry in volatility. This means that the impact of a positive market outcome on the future volatility is different from the impact of a negative outcome. Formally, this model parameterizes the conditional variance process as

\[ \ln(h_t) = \omega + \alpha g(z^*_t - 1) + \beta \ln(h_{t-1}) \] (5)

\[ g(z^*_t) = \theta z^*_t + \delta[|z^*_t| - \mathbb{E}[z^*_t]], \] (6)

where \( z^*_t = \epsilon_t / \sqrt{h_t} \) and \( \delta = 1 \). We refer to equations (2), (5), and (6) jointly as an EGARCH-M model.

The results of fitting the data using the three variations of the GARCH-M model are reported in Table 2. As is seen in Panel A, estimated GARCH parameters, namely \( \omega, \alpha \) and \( \beta \), are generally statistically different from zero. This observation may be viewed as support for existence of GARCH effects in the returns from Pacific basin markets, which is hardly surprising. However, the parameters of interest tell a different story. We are interested in estimated \( \mu \) and \( \gamma \)s. Merton’s specification of the risk-return relation implies that estimated \( \mu \)s should not be significantly different from zero. It is immediately obvious from Table 2 that is condition is met for almost all markets studied here. The exception are Colombia and Thailand. On the other hand, we also clearly observe that the second requirement of the theory, \( \gamma \gg 0 \), does not hold in the GARCH-M model. Together, these two results imply that GARCH(1,1)-in-Mean is not a good specification for measuring the risk-return trade-off in the Pacific basin markets.
Panel B in Table 2 reports the estimated parameters from fitting the EGARCH-M model to the data. Again, volatility parameters are generally statistically different from zero. Unlike the GARCH-M case, the majority of estimated intercept parameters in the risk-return relation are not statistically different from zero. Ten estimated intercepts are statistically non-zero, which means that using the EGARCH-M implies that the majority of these market face mild ICAPM inefficiency. The main problem with this specification is that while almost half of estimated $\gamma$s are statistically different from zero, all of them have a negative sign. In fact, only one market, Korea, demonstrates evidence of positive market price of risk. But the estimated $\gamma$ for Korea is not statistically significant. Again, we conclude that EGARCH-M may not be the best method for measuring risk-return relationship in the Pacific basin markets. Moreover, this conclusion implies that failure of GARCH-M model is not due to ignoring the leverage effect. EGARCH specification allows for leverage effect, but does not deliver a positive market price of risk.

Finally, Panel C of Table 2 reports the estimated parameters from fitting the Abs-GARCH-M model to the data. Similar to the two previous cases reported above, this specification fails to find a positive and statistically significant risk-return trade off in these markets. We conclude that symmetric GARCH models can not detect a positive risk-return relationship in the equity markets studied in this paper.

### 3.1.2 Rolling Window Measure of Volatility

An alternative to the parametric GARCH volatility measures studied in Section 3.1.1 is to construct non-parametric measures for volatility. In an influential paper, French et al. (1987) propose a rolling window estimator for market volatility. This method uses data sampled at the daily frequency to construct monthly volatility estimates. Following a modified version of the rolling sample approach in French et al. (1987), we estimate the conditional variance in equation (1) as:

$$V_{t}^{RW} = \frac{22}{D} \sum_{d=0}^{D} r_{t-d}^2$$

where $D$ is the number of days we use in the estimation. French et al. (1987) set the value of $D$ equal to 22 to obtain within-the-month measures of risk. Ghysels et al. (2005) allow for significantly longer windows, with $D$ set to more than 120 or over five months of trading. However, Ghysels et al. (2005) find that there seems to be an optimal $D$ size corresponding to four months of trading. We follow their example and set the value of $D = 88$. Thus, $V_{t}^{RW}$ corresponds to a scaled sum of daily squared returns going back 88 days.

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3We also estimated NGARCH-in-Mean results for each market’s excess returns, based on Engle and Ng (1993) NGARCH specification. These results are not qualitatively different from what is reported in Table 2. Thus, they are not reported. However, they are available upon request.
Using this constructed measure of monthly volatility, we estimate the parameters $\mu$ and $\gamma$ of the risk-return trade-off in equation (1). This strategy requires a two step procedure. We first construct the rolling window volatility measure, then we regress monthly excess returns on the constructed volatility series.

Panel A in Table 2 reports the regression results for the markets in our sample. As is immediately clear from this Table, most Pacific rim countries have either negative (nine markets) or positive but insignificant (four) trade-off relationships. Only Korean data supports a significantly positive risk-return trade-off.

### 3.1.3 MIDAS Measure of Volatility

Ghysels et al. (2005) examines the risk-return relationship using the mixed data sampling (MIDAS) method. The MIDAS estimator of the conditional variance $\text{Var}_t(r_{t+1})$ is based on historical squared returns:

$$V_{t}^{\text{MIDAS}} = 2 \sum_{d=0}^{\infty} w_d r_{t-d}^2$$

where $w_d$ is the weight assigned to the squared return $r_{t-d}^2$.

For the sake of parsimony, all the weights $w_d$ are postulated in a flexible function of two parameters $\kappa_1$ and $\kappa_2$ which control the speed of weights’ decay.

$$w_d = \frac{\exp(\kappa_1 d + \kappa_2 d^2)}{\sum_{i=0}^{\infty} \exp(\kappa_1 i + \kappa_2 i^2)}$$

The parameters $\kappa_1$, $\kappa_2$, $\mu$, and $\gamma$ are estimated by the maximum likelihood assuming that the distribution of the disturbance term in equation (1) conditioning on the variance estimator is normal.$^4$

Ghysels et al. (2005) apply the MIDAS method on the CRSP value-weighted U.S. stock returns and find empirical evidence supporting a robust, positive, and statistically significant market price of risk; hence empirical support for the fixed-parameter, linear form of ICAPM. The varying weights in equation (8) yield considerable flexibility in estimation of conditional variance, so that the risk-return relationship can be better assessed.

Panel B in Table 2 indicates a negative relationship between returns and volatility in case of eleven out of fifteen markets. Market returns in Korea, the Philippines, and Thailand, support a positive linear relationship with the MIDAS conditional variance. However, the estimated parameters are not statistically significant.

Figure ?? displays the MIDAS weights for all the Pacific-basin countries. We find that these weights decay at a much faster rate compared to the weights estimated using U.S. data and reported

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$^4$In our estimation, we impose the linear restriction that $\kappa_2 = 0$, following the instructions in Sinko et al. (2010). MATLAB code used for estimation is available from Eric Ghysels’ personal website.
by Ghysels et al. (2005). The exponential weights totally die out within two weeks in most of the countries we examined in this paper. This rapid decline in the importance of past daily squared returns is most likely responsible for the failure of the MIDAS method in capturing the expected positive market price of risk. We conducted extensive search over different weight and estimation configurations and feel confident that our results are robust to different estimation procedure options.

3.1.4 Volatility Regimes

According to Rossi and Timmermann (2009), the shape of the risk-return relationship may differ by the states of the economy. In order to explain this state dependent relationship between the conditional variance and return, we split the time series of the computed variances through MIDAS into two states:

\[
    r_t = \mu + \gamma_1 Var_{t-1}^+[r_t] + \gamma_2 Var_{t-1}^-[r_t] + \epsilon_t
\]

where

\[
    Var_{t-1}^+[r_t] = Var_{t-1}[r_t]1_{r_{t-1}>0} \\
    Var_{t-1}^-[r_t] = Var_{t-1}[r_t]1_{r_{t-1}<0}
\]

Panel C in Table 2 shows that more countries display a significant and positive risk-return tradeoff when there is a good realization of excess returns in the market \((r_{t-1} > 0)\). More precisely, Colombia, the Philippines, and Singapore show evidence of positive and significant response to good news. When bad news arrive in a market, only Korea still exhibits strong positive risk-return relationship. The response in Canada and Taiwan is statistically significant, but estimated \(\lambda_2\) is negative. We find additional evidences in support of the ICAPM as we sort the data in two market conditions. In order to further investigate the point that the relationship between risk and return may vary according to the market conditions, we also sorted the computed variances by the return quantile and only estimate the slope coefficients \((\gamma_1 \text{ and } \gamma_2)\) with the 10th – percentile and 90th – percentile of the returns. Empirically, these results confirm our previous findings that Philippines and Singapore have a significant positive risk-return trade-off with the 10% best returns. While Korea shows a very significant trade-off with the lowest 10% of the returns. However, due to the sparsity of data, these results need to be treated with caution. Hence, they are not reported here, but are available upon request.
3.2 Non-Zero Conditional Skewness in Returns: BiN-GARCH Measure of Volatility

In a recent paper, Feunou et al. (2010) question the two implicit underlying assumptions of the empirical risk-return literature, namely constant market price of risk and conditional symmetry in returns. As it is seen in Table 1, even unconditionally, the skewness of excess returns is significantly different from what you expect in a symmetric distribution such as normal or Student’s-t. Jondeau and Rockinger (2003) show that the existence of significant conditional negative skewness both in major international equity market and in foreign exchange market returns can not be ruled out. Thus, the assumption that conditional distribution of returns is symmetric, may be counterfactual.

On the other hand, Rossi and Timmermann (2009) question the assumption that market price of risk is a constant. Feunou et al. (2010) push this argument further and posit that market price of risk is not a constant parameter, but a time and state dependent process. They derive a closed form expression for this process in an endowment equilibrium economy populated with ambiguity averse agents.\(^5\)

Feunou et al. (2010) estimation procedure is based on a variation of GARCH which they call binormal GARCH (BiN-GARCH). This model assumes that excess returns are conditionally binormally distributed.\(^6\) Binormal distribution can be parameterized by the mean \(\mu_t\), the variance \(\sigma^2_t\), and the Pearson mode skewness, \(p_t\) as given by:\(^7\)

\[
\begin{align*}
\mu_t &= m_t + \sigma_t p_t \\
\sigma^2_t &= (1 - 2/\pi) (\sigma_{2,t} - \sigma_{1,t})^2 + \sigma_{1,t} \sigma_{2,t} \\
p_t &= \sqrt{2/\pi} (\sigma_{2,t} - \sigma_{1,t})/\sigma_t.
\end{align*}
\]

It can be shown that the initial parameters \(\sigma_{1,t}\) and \(\sigma_{2,t}\) are expressed in terms of the total variance and the Pearson mode skewness as follows:

\[
\begin{align*}
\sigma_{1,t} &= \sigma_t \left( -\sqrt{\pi/8} p_t + \sqrt{1 - (3\pi/8 - 1) p^2_t} \right) \\
\sigma_{2,t} &= \sigma_t \left( \sqrt{\pi/8} p_t + \sqrt{1 - (3\pi/8 - 1) p^2_t} \right).
\end{align*}
\]

Feunou et al. (2010) show that in their equilibrium model, the conditional mode is an implicit nonlinear function of conditional downside and upside volatilities, \(m_t = g(\sigma_{1,t}, \sigma_{2,t})\). The implicit

\(^5\)They use a recursive form of Gul (1991) disappointment aversion preferences. For examples and detailed discussions of this recursive utility class, refer to Routledge and Zin (2010) and Bonomo et al. (2011).

\(^6\)Binormal distribution has a history in natural sciences. It was first introduced by Gibbons and Mylroie (1973). Feunou et al. (2010) are the first to use this distribution in a financial application. Examples from other branches of science include Bangert et al. (1986), Kimber and Jeynes (1987), and Garvin and McClean (1997).

\(^7\)Pearson mode skewness is defined as \((\mu_t - m_t)/\sigma_t\).
function $g$ is parameterized by the preference parameters. They provide a detailed discussion in their paper and an external appendix. The equation $m_t = g(\sigma_{1,t}, \sigma_{2,t})$ defines a new risk-return relation that relates the conditional mode to the conditional downside and upside volatilities. To be able to deal with this new trade-off between risk and reward, they first-order linearize the nonlinear model around the steady state values ($\bar{\sigma}_1, \bar{\sigma}_2$) to obtain

$$m_t = g(\sigma_{1,t}, \sigma_{2,t}) \approx \lambda_0 + \lambda_1 \sigma_{1,t} + \lambda_2 \sigma_{2,t}. \quad (16)$$

Given the expression (16), the traditional risk-return trade-off that relates expected returns to the total variance may be expressed as:

$$\mu_t = m_t + \sigma_t \quad (17)$$

$$E_t[r_{t+1}] = \lambda_0 + \lambda^*_t \sigma_t \quad (18)$$

where

$$\lambda^*_t = \left(1 - (\lambda_1 - \lambda_2) \sqrt{\pi/8}\right) p_t + (\lambda_1 + \lambda_2) \sqrt{1 - (3\pi/8 - 1) p_t^2}. \quad (19)$$

The first equality in Eq. (17) follows by the definition of mean in binormal distribution, Eq. (11). The second equality in Eq. (17) and Eq. (19) follow from Eq. (14) and Eq. (16). Eq. (17) characterizes the traditional risk-return trade-off in this model, and shows that the price of risk depends on the asymmetry in returns.

In equilibrium, expression (16) holds exactly:

$$m_t = \lambda_0 + \lambda_1 \sigma_{1,t} + \lambda_2 \sigma_{2,t}. \quad (20)$$

If $\lambda_2 \approx -\lambda_1$, then the mode is a function of the relative downside volatility, $\sigma_{1,t} - \sigma_{2,t}$, that is

$$m_t \approx \lambda_0 + \lambda_1 (\sigma_{1,t} - \sigma_{2,t}) \quad (21)$$

and the price of risk in the traditional risk-return trade-off simplifies to:

$$\lambda^*_t = \left(1 - \lambda_1 \sqrt{\pi/2}\right) p_t. \quad (22)$$

Feunou et al. (2010) show that the coefficients $\lambda_0$, $\lambda_1$ and $\lambda_2$ all depend on preference parameters. There are three main model implications for the risk-return trade-off. First, the loading of the conditional mode on downside volatility, $\lambda_1$, is positive, implying that the conditional mode increases to compensate for an increase in downside volatility. Second, the loading of the conditional mode on upside volatility, $\lambda_2$, is negative, which implies that the conditional mode increases
to compensate for a decrease in upside volatility. Third, \( \lambda_2 \) is very close to \(-\lambda_1\). We have discussed this case in a previous paragraph and subsequently show that this restriction is statistically supported by the data. Thus, only the relative downside volatility, \( \sigma_{1,t} - \sigma_{2,t} \), seems to matter in equilibrium. An increase in relative downside volatility is compensated with an increase in the conditional mode, \( m_t \).

To close the model, we need to characterize the dynamics of Pearson mode skewness, \( p_t \), and total volatility, \( \sigma_t \). We follow Feunou et al. (2010) and assume that \( \sigma_t \) follows the nonlinear GARCH (NGARCH) dynamics of Engle and Ng (1993). This assumption implies that

\[
\sigma_{t+1}^2 = \beta_0 + \beta_1 \sigma_t^2 + \beta_2 \sigma_t^2 (z_{t+1} - \theta)^2, \tag{23}
\]

where \( z_{t+1} = (r_{t+1} - \mathbb{E}_t [r_{t+1}]) / \sigma_t \) are standardized residuals. BiN-GARCH model nests NGARCH. The only requirement is \( \sigma_{1,t} = \sigma_{2,t} \).

We directly follow Feunou et al. (2010) and assume that Pearson mode skewness dynamics are characterized by:

\[
p_{t+1} = \sqrt{\frac{2}{\pi - 2}} \tanh (\kappa_0 + \kappa_1 z_{t+1}^* I (z_{t+1}^* \geq 0) + \kappa_2 z_{t+1}^* I (z_{t+1}^* < 0) + \kappa_3 p_t), \tag{24}
\]

where \( z_{t+1}^* = (r_{t+1} - m_t) / \sigma_t \). This nonlinear GARCH-type dynamics of the conditional Pearson mode skewness also features asymmetry in asymmetry. Asymmetries in the Pearson mode skewness are generated by deviations of realized returns from the conditional mode. This formulation is based on the autoregressive conditional skewness of Harvey and Siddique (1999, 2000).

Table 4 reports maximum likelihood estimation results for fitting the unrestricted BiN-GARCH model in Eq. (20) to excess returns data. Returns data from eight markets supports statistically significant and positive \( \lambda_1 \) and negative \( \lambda_2 \). Two markets, Taiwan and Thailand, deliver estimated parameters that have the expected sign, but are not statistically significant. The remaining four markets, Canada, Colombia, Mexico, and Malaysia do not support the positive loading on downside and negative loading on upside volatility.

For the majority of markets that support positive \( \lambda_1 \) and negative \( \lambda_2 \), the following regularities are noticeable: First, absolute values of the estimated \( \lambda_1 \) and \( \lambda_2 \) are very close, generally less than one standard error apart. Second, unconditional expected values of Pearson mode skewness, \( \mathbb{E}(p_t) \), are negative and thus lend additional credibility to negative conditional skewness in excess returns. Third, for all markets studied, BiN-GARCH is preferable to NGARCH model, based on the reported likelihood ratio (LR) test statistics reported in the last row of the table.

Based on what is observed in Table 4, we find it reasonable to impose the linear restriction \( \lambda_1 =
−λ_2 and estimate the relationship between the conditional mode and the relative downside risk, Eq. (21). Maximum likelihood estimation results from fitting this equation to data are presented in Table 5. It is immediately obvious that estimated λ_1 parameter is statistically significant and positive for nine markets studied in this paper. It is negative, but statistically significant for Colombia. Given this restricted formulation, we can compute the expected traditional market price of risk, Eq. (22), easily. These results, denoted E(λ^*_t), imply that in eight markets in the sample, data supports a positive market price of risk. This observation translates into over 50% success in capturing positive risk-return tradeoff in the cross section of the sample. This ratio is larger than any reasonable assumption about observed variation in the cross section under the null of no positive risk-return tradeoff. Notice that by this measure, BiN-GARCH is far more successful than all the other models discussed in the paper in capturing the positive market price of risk in the Pacific basin markets. Similar to what is observed for the unrestricted BiN-GARCH model, the restricted version is preferable to NAGARCH benchmark model, as attested by the LR test statistics in the last row of the table.

4 Conclusions

We study risk-return tradeoff relationship in fourteen Pacific basin financial markets. The main question addressed in the paper is whether modeling, and pricing the premia, for the first two moments is adequate for capturing the dynamics of risk-return tradeoff relationship. We empirically demonstrate that in addition to conditional mean and volatility, one needs to model the conditional skewness.

We show that regardless of the returns volatility model used, assuming zero conditional skewness which means symmetry in returns, leads to failure in capturing the expected positive market price of risk. We also empirically demonstrate that allowing for time-varying and non-zero conditional skewness in BiN-GARCH model, we successfully capture a positive market price of risk in over half of the markets studies.

These findings have an important impact on the way we study risk-reward relationship. We show that market price of risk modeled as a time-varying process, and not a constant parameter, is better supported by the data in Pacific basin markets. This implies that the level of effective risk tolerance, following Merton (1973) formulation, is time-varying. In turn, this observation has profound implications for financial activities such as portfolio choice and option pricing that make implicit or explicit assumptions regarding risk tolerance of market participants. On the other hand, we show that ignoring conditional skewness leads to considerable misspecification in the econometric models of risk. Thus, to construct more accurate risk management tools such as Value-at-Risk (VaR) or expected shortfall (ES), a careful modeling of conditional higher moments, such as conditional skewness, is crucial.
References


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<td>-0.92</td>
<td>-0.38</td>
</tr>
<tr>
<td>Std. Dev. (%)</td>
<td>25.24</td>
<td>20.07</td>
<td>33.22</td>
<td>35.84</td>
<td>49.15</td>
<td>21.89</td>
<td>37.93</td>
<td>29.95</td>
<td>31.29</td>
<td>23.58</td>
<td>32.40</td>
<td>37.44</td>
<td>40.02</td>
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</tr>
<tr>
<td>Skewness</td>
<td>-1.62</td>
<td>-1.04</td>
<td>-0.42</td>
<td>-0.54</td>
<td>0.19</td>
<td>0.00</td>
<td>0.20</td>
<td>-0.24</td>
<td>-1.03</td>
<td>-0.34</td>
<td>-0.15</td>
<td>-0.56</td>
<td>-0.07</td>
<td>-0.52</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>10.86</td>
<td>3.78</td>
<td>-0.93</td>
<td>6.78</td>
<td>4.53</td>
<td>0.54</td>
<td>2.87</td>
<td>4.04</td>
<td>3.26</td>
<td>1.32</td>
<td>1.89</td>
<td>6.01</td>
<td>1.43</td>
<td>2.19</td>
</tr>
<tr>
<td>Start Date</td>
<td>02-70</td>
<td>02-70</td>
<td>01-93</td>
<td>02-70</td>
<td>01-88</td>
<td>02-70</td>
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<td>02-70</td>
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<tr>
<td>No. Obs.</td>
<td>470</td>
<td>470</td>
<td>194</td>
<td>470</td>
<td>254</td>
<td>470</td>
<td>254</td>
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<td>254</td>
<td>254</td>
<td>470</td>
<td>254</td>
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</table>

This table reports the descriptive statistics for the data used in this study. The reported mean and standard deviations are in annualized percentages. We report excess skewness and kurtosis for the data. These descriptive statistics reflect monthly sampled MSCI country index excess returns data, ending in February 27, 2009. We use U.S. 3-month T-Bill rate as our proxy for the risk-free rate. Source: Thomson Reuters Datastream.
Table 2: GARCH Estimation Results

<table>
<thead>
<tr>
<th>Country</th>
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<th>Phil</th>
<th>Sing</th>
<th>Taiwan</th>
<th>Thai</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>0.5002</td>
<td>0.8312</td>
<td>0.7107</td>
<td>1.0187</td>
<td>2.3110</td>
<td>0.1149</td>
<td>0.4255</td>
<td>1.1231</td>
<td>2.3498</td>
<td>1.9926</td>
<td>2.7512</td>
<td>0.5602</td>
<td>0.5067</td>
<td>2.5176*</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>(0.7559)</td>
<td>(0.729)</td>
<td>(0.6000847)</td>
<td>(0.64609)</td>
<td>(0.1614)</td>
<td>(0.1035)</td>
<td>(0.1026)</td>
<td>(0.5667)</td>
<td>(1.17657)</td>
<td>(1.4278)</td>
<td>(1.7122)</td>
<td>(0.4999)</td>
<td>(1.2821)</td>
<td>(1.0068)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>(0.0067)</td>
<td>(0.0103)</td>
<td>3.57E-05</td>
<td>0.0070</td>
<td>-0.0056</td>
<td>0.0060</td>
<td>0.0036</td>
<td>-0.0059</td>
<td>0.0080</td>
<td>-0.0396</td>
<td>-0.0262</td>
<td>-0.0035</td>
<td>-0.0028</td>
<td>-0.0162</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.9528</td>
<td>(2.6592)</td>
<td>7.99</td>
<td>(4.7755)</td>
<td>1.97</td>
<td>(5.8875)</td>
<td>(1.9505)</td>
<td>(4.6154)</td>
<td>(2.2255)</td>
<td>(3.6311)</td>
<td>(7.2177)</td>
<td>(1.7900)</td>
<td>(3.7575)</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: EGARCH(1,1)-in-Mean

| Log-Sig | 1533.8101 | 1426.79 | 1442.39 | 1672.58 | 1106.937 | 1477.452 | 937.06 | 860.75 | 927.94 | 840.06 | 920.74 | 1518.70 | 948.85 | 955.51 |
| SDC | 3098.22 | 2884.18 | 713.30 | 3375.75 | 2056.02 | 2985.53 | 1903.69 | 1749.19 | 1883.57 | 1707.80 | 1869.17 | 3139.93 | 1925.38 | 1938.70 |

Panel C: EGARCH(1,1)-in-Vol

This Table reports estimated parameter values for three variants of GARCH-in-Mean model. Standard errors appear in parentheses. † and * denote rejection of the null hypothesis that the parameter equals zero at the 10% and 5% significance levels, respectively. The estimated parameters were obtained by maximum likelihood. In each case, the conditional mean equation is given by \( r_{t} = \mu + \gamma h_{t-1} + \epsilon_{t} \). \( r_{t} \) is the market excess return in country \( i \). \( \epsilon_{t} \sim N(0,1) \). \( h_{t} \) is the conditional variance in market \( i \). We use MSCI country index excess returns data, ending in February 27, 2009. We use U.S. 3-month T-Bill rate as our proxy for the risk-free rate. Source: Thomson Reuters Datastream.
Table 3: Rolling Window, MIDAS, and Sign-Conditioned Volatility Estimation Results

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<tr>
<th>Country</th>
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<th>Canada</th>
<th>Col</th>
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<th>Japan</th>
<th>Korea</th>
<th>Malay</th>
<th>Mexico</th>
<th>NZ</th>
<th>Phil</th>
<th>Sing</th>
<th>Taiwan</th>
<th>Thai</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>0.5528</td>
<td>0.8431*</td>
<td>1.2895</td>
<td>0.7348</td>
<td>0.3762</td>
<td>0.217</td>
<td>-1.0205</td>
<td>0.3178</td>
<td>2.6422</td>
<td>1.6347</td>
<td>-0.7735</td>
<td>0.2817</td>
<td>1.8969</td>
<td>1.2837</td>
</tr>
<tr>
<td>(0.5485)</td>
<td>(0.3150)</td>
<td>(1.1887)</td>
<td>(1.2065)</td>
<td>(1.1782)</td>
<td>(0.9255)</td>
<td>(0.911)</td>
<td>(0.8413)</td>
<td>(1.0121)</td>
<td>(1.1404)</td>
<td>(1.3836)</td>
<td>(1.0159)</td>
<td>(1.1545)</td>
<td>(1.1242)</td>
<td></td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-0.0407</td>
<td>-0.1661†</td>
<td>-0.0979</td>
<td>0.0199</td>
<td>0.0519</td>
<td>0.1019†</td>
<td>-0.0037</td>
<td>-0.1750</td>
<td>-0.3966</td>
<td>0.1542</td>
<td>0.0180</td>
<td>-0.2055</td>
<td>-0.1346</td>
<td></td>
</tr>
<tr>
<td>(0.1030)</td>
<td>(0.0751)</td>
<td>(0.1569)</td>
<td>(0.0489)</td>
<td>(0.1949)</td>
<td>(0.0514)</td>
<td>(0.096)</td>
<td>(0.1017)</td>
<td>(0.2179)</td>
<td>(0.1949)</td>
<td>(0.1083)</td>
<td>(0.1083)</td>
<td>(0.0925)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \kappa )</td>
<td>-2.1707*</td>
<td>-0.0127</td>
<td>-0.2468</td>
<td>-1.5383</td>
<td>0.0180</td>
<td>-0.2055</td>
<td>-0.1346</td>
<td>-0.3645</td>
<td>-0.2913*</td>
<td>0.0078</td>
<td>(0.6847)</td>
<td>(0.2613)</td>
<td>(1.1136)</td>
<td>(0.4676)</td>
</tr>
<tr>
<td>(0.1022)</td>
<td>(0.0750)</td>
<td>(0.1696)</td>
<td>(0.1573)</td>
<td>(0.0534)</td>
<td>(0.1950)</td>
<td>(0.0522)</td>
<td>(0.0964)</td>
<td>(0.1023)</td>
<td>(0.2175)</td>
<td>(0.2015)</td>
<td>(0.2162)</td>
<td>(0.1152)</td>
<td>(0.1128)</td>
<td></td>
</tr>
</tbody>
</table>

Panel A: Rolling Windows with Equal Weights

Panel B: MIDAS

Panel C: Volatility Conditioned on \( \text{sign}(r_{t-1}) \)

This Table reports estimated parameter values for three volatility estimation specifications. Standard errors appear in parentheses. † and * denote rejection of the null hypothesis that the parameter equals zero at the 10% and 5% significance levels, respectively. The estimated parameters were obtained by maximum likelihood. In each case, the conditional mean equation is given by:

\[ r_i^t = \mu + \gamma_1 V_{t-1}^{-}[r_t] + \varepsilon_t \]

where \( \varepsilon_t \sim N(0,1) \). \( V_{t-1}^{-}[r_t] \) is the conditional variance in market \( i \), and is specified as French et al. (1987) rolling estimator, \( V_{t-1}^{-}[r_t] = \sum_{d=0}^{D} r_{t-d}^2 \), in Panel A and by Ghysels et al. (2005) MIDAS estimator, \( V_{t-1}^{-}[r_t] = \sum_{d=0}^{\infty} w_d r_{t-d}^2 \) where \( w_d, \kappa_1 \) and \( \kappa_2 \) are as in Eq. (9), in Panel B. In Panel C, we split the computed MIDAS conditional variances depending on whether \( r_{t-1} \) is positive or negative. We estimate \( r_t = \mu + \gamma_1 V_{t-1}^{+}[r_t] + \gamma_2 V_{t-1}^{-}[r_t] + \varepsilon_t \). We use MSCI country index excess returns data, ending in February 27, 2009. We use U.S. 3-month T-Bill rate as our proxy for the risk-free rate. Source: Thomson Reuters Datastream.
Table 4: Full BiN-GARCH Estimation Results

<table>
<thead>
<tr>
<th>Country</th>
<th>Aus</th>
<th>Canada</th>
<th>Col</th>
<th>HK</th>
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<th>Sing</th>
<th>Taiwan</th>
<th>Thai</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0$</td>
<td>0.0005</td>
<td>$(0.0005)$</td>
<td>0.0007</td>
<td>$(0.0007)$</td>
<td>0.0009†</td>
<td>$(0.0009)$†</td>
<td>0.0010‡</td>
<td>$(0.0010)$‡</td>
<td>0.0012</td>
<td>$(0.0012)$</td>
<td>0.0008</td>
<td>$(0.0008)$</td>
<td>0.0004</td>
<td>$(0.0004)$</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.5012*</td>
<td>$(0.0553)$</td>
<td>0.5864</td>
<td>$(0.4445)$</td>
<td>0.1272</td>
<td>$(0.1415)$</td>
<td>0.3344*</td>
<td>$(0.0606)$</td>
<td>0.4094*</td>
<td>$(0.0642)$</td>
<td>0.4692*</td>
<td>$(0.0876)$</td>
<td>0.3619†</td>
<td>$(0.1556)$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>-0.4842*</td>
<td>0.8684</td>
<td>0.2168</td>
<td>0.3259*</td>
<td>-0.3794*</td>
<td>-0.3903*</td>
<td>-0.3508</td>
<td>0.1412</td>
<td>0.2205</td>
<td>-0.4074</td>
<td>-0.2802</td>
<td>-0.1820†</td>
<td>-0.0924</td>
<td>-0.2627</td>
</tr>
<tr>
<td>$E(p_t)$</td>
<td>-0.1166</td>
<td>-0.0946</td>
<td>0.0162</td>
<td>-0.1087</td>
<td>0.0810</td>
<td>0.0400</td>
<td>0.0152</td>
<td>-0.0620</td>
<td>-0.0445</td>
<td>-0.0782</td>
<td>-0.0688</td>
<td>-0.0011</td>
<td>-0.0091</td>
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<tr>
<td>Log-Lik</td>
<td>23,464.38</td>
<td>25,942.81</td>
<td>13,026.92</td>
<td>22,060.54</td>
<td>91</td>
<td>15,069.88</td>
<td>23,067.50</td>
<td>14,879.15</td>
<td>17,286.56</td>
<td>15,611.85</td>
<td>16,753.38</td>
<td>15,883.84</td>
<td>23,948.82</td>
<td>15,060.26</td>
</tr>
<tr>
<td>LR Stat</td>
<td>73.01</td>
<td>26.27</td>
<td>141.33</td>
<td>143.15</td>
<td>48.85</td>
<td>137.29</td>
<td>264.44</td>
<td>26.00</td>
<td>104.91</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

This Table reports estimated parameter values for the full BiN-GARCH model of Feunou et al. (2010). Standard errors appear in parentheses. †, ‡ and * denote rejection of the null hypothesis that the parameter equals zero at the 10%, 5%, and 1% significance levels, respectively. The estimated parameters were obtained by maximum likelihood. $\lambda_0$, $\lambda_1$, and $\lambda_2$ are obtained from fitting $m_t = \lambda_0 + \lambda_1 \sigma_{t-1} + \lambda_2 \sigma_{2,t}$ for each series. $E(p_t)$ is the unconditional mean of $p_t = \sqrt{2/(2 - \pi)} \tanh (\delta_0 + \delta_1 z_t^t (I(z_t^t \geq 0) + \delta_2 z_t^t (I(z_t^t < 0) + \delta_3 p_{t-1})$ process, where $z_t^t = (r_t - m_{t-1})/\sigma_{t-1}$. We use MSCI country index excess returns data, ending in February 27, 2009. We use U.S. 3-month T-Bill rate as our proxy for the risk-free rate. Source: Thomson Reuters Datastream.
Table 5: BiN-GARCH Estimation Results with Relative Downside Risk

<table>
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<th>Sing</th>
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<th>Thai</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0$</td>
<td>0.0006</td>
<td>0.0010</td>
<td>1.55E-05</td>
<td>0.0010</td>
<td>5.45E-06</td>
<td>0.0001</td>
<td>0.31E-05</td>
<td>0.0006</td>
<td>0.0014</td>
<td>0.0005</td>
<td>0.0006</td>
<td>-6.60E-05</td>
<td>4.23E-05</td>
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</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.0985</td>
<td>-0.1494</td>
<td>-0.2725</td>
<td>0.3255</td>
<td>0.3477</td>
<td>0.1573</td>
<td>-0.0074</td>
<td>0.3221</td>
<td>0.2251</td>
<td>0.0761</td>
<td>-0.2987</td>
<td>0.2598</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(p_t)$</td>
<td>-0.1166</td>
<td>-0.0958</td>
<td>0.0162</td>
<td>-0.1087</td>
<td>0.0808</td>
<td>0.0032</td>
<td>0.0146</td>
<td>-0.0591</td>
<td>-0.0474</td>
<td>-0.0564</td>
<td>0.0205</td>
<td>0.0077</td>
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<td></td>
</tr>
<tr>
<td>Log-Lik</td>
<td>23464.32</td>
<td>25934.19</td>
<td>13026.17</td>
<td>22060.52</td>
<td>15069.46</td>
<td>23065.94</td>
<td>14879.14</td>
<td>17273.96</td>
<td>15609.96</td>
<td>16752.86</td>
<td>23947.83</td>
<td>15060.02</td>
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<tr>
<td>LR Stat.</td>
<td>35.94</td>
<td>77.67</td>
<td>78.60</td>
<td>203.64</td>
<td>236.66</td>
<td>69.90</td>
<td>26.26</td>
<td>116.12</td>
<td>139.38</td>
<td>47.79</td>
<td>135.20</td>
<td>262.46</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This Table reports estimated parameter values for the BiN-GARCH model of Feunou et al. (2010). Standard errors appear in parentheses. † and * denote rejection of the null hypothesis that the parameter equals zero at the 10% and 5% significance levels, respectively. The estimated parameters were obtained by maximum likelihood. $\lambda_0$ and $\lambda_1$ are obtained from fitting $m_t = \lambda_0 + \lambda_1 (\sigma_1, t - \sigma_2, t) + \epsilon_t$ for each series. $E(p_t)$ is the unconditional mean of $p_t = \sqrt{2/(2 - \pi)} \tan(\delta_0 + \delta_1 z_t^* (I(z_t^* \geq 0) + \delta_2 z_t^* (I(z_t^* < 0) + \delta_3 p_{t-1})$ process, where $z_t^* = (r_t - m_{t-1}) / \sigma_{t-1}$. $E(\gamma_t)$ is the unconditional mean of the market price of risk process, $\gamma_t = p_t (1 - \lambda_1 \sqrt{\pi/2})$. We use MSCI country index excess returns data, ending in February 27, 2009. We use U.S. 3-month T-Bill rate as our proxy for the risk-free rate. Source: Thomson Reuters Datastream.
Figure 1: MIDAS Weights for Individual Equity Market.

This figure plots the MIDAS weights in 15 markets investigated in this study. The figure displays the weights estimated from the entire sample, and based on daily data used for construction of monthly MIDAS volatilities. The weights are calculated by substituting the estimated values of $\kappa_1$ (we impose the linear restriction $\kappa_2 = 1 - \kappa_1$) from daily MIDAS into the weight function Equation (9). The estimates of $\kappa_1$ are shown in Table 3.