1

INTRODUCTION TO PROBABILITY

“Builds upon knowledge of probability first learned in seventh grade that includes uses of random sampling to draw inferences, compares inferences between populations, investigates chance processes, and develops, uses and evaluates probability models.

The unit introduces foundational properties and rules of probabilities, starting with events as subsets of sample space and introducing independence and conditional probability. The unit includes treatment of rules of probability to determine likelihood of compound events and introduces counting principals to calculate probability.”

-Jeff Merithew
This unit will present basic probability concepts to students along with incorporating mathematical formulas used to solve problems. Use this video to start the lesson and get the students thinking about how probability applies to the real world.

The following is a PRE-ASSESSMENT for your students prior to starting the unit.
This is an introduction to the fundamental concepts of probability. In each section you will have content objectives, essential questions and additional activities to help students demonstrate an understanding of probability.
Objectives:

S-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or”, “and”, “not”).

Students will...

• Collect data through surveys, observational studies and experiments and analyze the results.

• Make a sample space for selected experiments and represent it in the form of a list, chart, picture, or tree diagram.
Essential Questions:

How can an event be described as a subset of outcomes using correct set notation?

How can you determine possible outcomes for an experiment?

Procedure (The lesson is structured in the following way):

1. Begin the lesson by introducing important vocabulary:
   Sample Space; Event; Tree Diagram; Fundamental Principle of Counting

2. To help students understand sample space roll a die and flip a coin. Have students list all of the possible outcomes—this is their sample space. You can then define an event such as “rolling an even number on a dice” or “getting a head on the coin”. Introduce set notation.

3. Have students create charts or diagrams as a visual representation of all of the possible outcomes, i.e. tree diagrams and venn diagrams. Below is an example of a venn diagram.

   In a class of 50 students, 18 take chorus, 26 take band, and 2 take chorus and band. Create a venn diagram to display the information.

   Use the following template to have students create their own Venn Diagrams.

   Venn Diagram template
4. Attached (*to the right*) is a lesson plan for use with students which incorporates activities with tree diagrams.

5. Close with the Fundamental Counting Principle as a way to determine the number of outcomes in a sample space without making a list.

6. Practice Assessments

![Practice with Venn Diagrams](image1)

![Working with Tree Diagrams](image2)

This notepad is placed at the end of each Section to provide you with the opportunity to record your thoughts on the lesson and reference at a later date.
Objectives:

**S-CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

**S-CP.7** Apply the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), and interpret the answer in terms of the model.

Students will...

- Identify the sample space and subsets of an event, as well as use basic probability rules to solve problems to include unions, intersections, complementary events and mutually exclusive events.

- Determine and interpret the probability of an event occurring from a given sample space and represent the probability as a ratio decimal or percent, as appropriate for the given situation.

- Understand that two events A and B are mutually exclusive if and only if \( P(A \text{ and } B) = 0 \).

- Determine whether two events are disjoint (mutually exclusive).
Essential Questions:
How are probabilities, including compound probabilities, calculated?

Procedure (The lesson is structured in the following way):

1. Begin the lesson by introducing important vocabulary: Intersection; Union; Mutually Exclusive Events; Complementary Events

2. From the previous lesson students have practiced listing all of the outcomes so refer back to one of those examples. You can then discuss the concept of assigning a probability to the outcomes and to each of the events based on the sample space.

3. Introduce essential formulas using the activity sheet (to the right) that need to be used in this section, i.e. basic probability rules and formulas, as well as the addition rule.
4. Have students use charts and diagrams to calculate probabilities for compound events

5. Discuss the relationship between the definition of mutually exclusive and the formula to prove $P(A \text{ and } B) = 0$
**Objectives:**

**S-CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Students will...

- Create a two-way frequency table from a set of data on two categorical variables.
- Determine if two categorical variables are independent by analyzing a two-way table of data collected on the two variables.
- Calculate conditional probabilities based on two categorical variables and interpret in context.
Essential Questions:
How are two-way frequency tables used to model real-life data?

Procedure *(The lesson is structured in the following way):*

1. Begin with a review of all probability rules from past lessons.

2. Using a given two-way frequency table determine probabilities of single events and compound events. Then proceed with the attached interactive activity *(below)* with students to practice completing a table, as well as determining outcomes.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Preferred Program</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dance</td>
</tr>
<tr>
<td>Women</td>
<td>16</td>
</tr>
<tr>
<td>Men</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>18</td>
</tr>
</tbody>
</table>

3. Follow up by having students collect data and construct their own two-way frequency table, as well as calculate specific probabilities.
OBJECTIVES:

S-CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Students will...

- Understand two events A and B are independent if and only if \( P(A \text{ and } B) = P(A) \times P(B) \).

- Given an everyday situation describing two events, use the context to construct an argument as to whether the events are independent or dependent.
Essential Questions:

How are probabilities, including compound probabilities, of independent events calculated?

How are probabilities of independent events compared to their compound probabilities?

Procedure (The lesson is structured in the following way):

1. Begin the lesson by introducing important vocabulary:
   Independent Events; Dependent Events

   Attached (to the right) is a sample lesson plan (with videos) to use for distinguishing the differences between the 2 types of events.

2. Give students a list of sets of events and have them determine if the events are independent or dependent.

   **Question 1 of 5**
   Drawing two red marbles from a bag without replacement.

   - A. Independent Events
   - B. Dependent Events
3. Next, introduce essential formula that will be used in this section: 
\[ P(A \text{ and } B) = P(A) \times P(B). \]

4. Have students use given probability information to tell if two events are independent using the formula.

Revisit the previous sample lesson plan *(to the right)* to access sample problems.

5. Lastly, have students use their created two-way table (from previous section) to tell if two events are independent based on the formula.

6. Practice Assessment

![Practice Assessment](image-url)
Conditional Probability refers to calculating the likelihood that an event will occur when a condition has been placed that another event has already occurred. In some cases, when two events are not independent, the condition changes the probability of the second event.

For example, what is the probability that the family has 2 female children versus what is the probability that the 2nd child born to a family is a girl knowing the 1st child was a girl?
Objectives:

S-CP.3 Understand the conditional probability of A given B as \( P(A \cap B)/P(B) \), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S-CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

S-CP.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.

Students will:

• Understand that the conditional probability of event A given event B has already happened is given by the formula: \( P(A|B) = \frac{P(A \cap B)}{P(B)} \).

• Understand that when finding the conditional probability of A given B, the sample space is reduced to the possible outcomes for event B. Therefore the probability of event A happening is the fraction of event B’s outcomes that also belong to A.
Essential Questions:

How are conditional probabilities calculated using information in a variety of formats?

How are conditional probabilities and independence interpreted in relation to a situation?

Procedure (The lesson is structured in the following way):

1. Begin by explaining how up until this point we have discussed finding the probability that an event will happen. Now we are going to examine the probability an event will occur when we know that another event has already occurred. Close this introduction of the lesson by defining Conditional Probability.

2. Next, introduce the formula for conditional probability.

\[ P(B|A) = \frac{P(A \cap B)}{P(A)} \]
3. You can utilize students previous knowledge of Venn Diagrams (from Ch. 2 Sec. 1) to illustrate how when you are talking about a single event occurring then you are focusing on only one of the circles. However, when you include the fact that the other event is occurring as well, then you are referencing the intersection out of that previously single event. This is the reason for having all the different probabilities involved in this one formula.

For student familiarity, you can use the sample Venn Diagram from Ch. 2 Sec.1.

**Brief Explanation of Conditional Probability**
*(as it relates to the Venn Diagram Above)*

The probability of a student being in chorus given that they are in band would mean you are only focused on the 26 band students and the fact that 2 of those students are in also taking chorus.
4. Now that you have connected conditional probability to Venn Diagrams, use the activity sheet *(to the right)* to build a connection between two way frequency tables and conditional probability.

**Note:** The Let’s Make A Deal Applet *(included on page 2 of the attached activity sheet)* is great to use as a game to demonstrate conditional probability. It does require Java *(or a Java plug-in)* to open on Mac or PC devices.

If you are unable to open the activity sheet *(attached above)*,

Click [HERE](#) to access the Let’s Make a Deal Applet

*Source: gamesmuseum.uwaterloo.ca*
5. Attached below are additional Game Assessments for students to use to practice, or for you (the teacher) to use as formative assessments of Conditional Probability.

**Note:** You can differentiate these assessments for any student who is having difficulty understanding Conditional Probability (or simply needs more practice before completing the assessment) by switching to the Non-Game Version once you access the website (refer to the menu located at the top left corner of the page). The Non-Game Version allows a student to get help with questions without effecting their overall score for the assessment.
Objectives:

**S-CP.8** Apply the general Multiplication Rule in a uniform probability model, \( P(A \text{ and } B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B) \), and interpret the answer in terms of the model.

Students will:

- Calculate probabilities using the general multiplication rule and interpret in context.
- Understand two events A and B are independent if and only if \( P(A|B) = P(A) \) and \( P(B|A) = P(B) \).
**Essential Questions:**

How are conditional probabilities calculated using information in a variety of formats?

How are conditional probabilities and independence interpreted in relation to a situation?

**Procedure (The lesson is structured in the following way):**

1. Refer back to the formula for Conditional Probability (from Sec. 1) and solve for the probability of the intersection algebraically. This “new” formula is known as the General Multiplication Rule: 
   \[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]

2. Using the fact that if two events are independent, then the probability that B happens given A has already happened does not change the probability of B; therefore \( P(B|A) = P(B) \). Using substitution, replace \( P(B|A) \) with \( P(B) \) in the formula you created in Step 1 to derive the General Multiplication Rule that students will use when two events are proved to be independent: 
   \[ P(A \text{ and } B) = P(A) \cdot P(B) \]

3. Attached (to the right) is a tutorial that you (or the students) can use to help develop a connection between Conditional Probability, the General Multiplication Rule and Independent Events.
Counting techniques can be used to find all the possible ways to complete different tasks or choose items from a list. (Pearson, Geometry Teacher Edition p. 836)

Is there a difference between the number of outcomes when order is not a factor versus when order matters?

4

PERMUTATIONS AND COMBINATIONS

Counting techniques can be used to find all the possible ways to complete different tasks or choose items from a list. (Pearson, Geometry Teacher Edition p. 836)

Is there a difference between the number of outcomes when order is not a factor versus when order matters?
Objective:

S-CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

Students will:

- Identify situations as appropriate for use of a permutation or combination to calculate probabilities.
- Use permutations and combinations in conjunction with other probability methods to calculate probabilities of compound events and solve problems.
Essential Questions:

What is the difference between a permutation and a combination?

Which one has more outcomes?

Procedure *(The lesson is structured in the following way)*:

1. Begin the lesson by having five students come to the front of the room and see if the class can determine how many different ways you could arrange three of the five students. If you find that students are repeating ways to organize, encourage them to use some type of organization (i.e., lists, tree diagrams, table, etc.).

2. Attached *(below)* is a small group activity to use in addition to your whole group introduction of the lesson *(or in replace of this activity)* for students to investigate how to determine all the possible outcomes for a given situation.
3. Once they have determined their answer for the introduction activity introduce the idea of whether or not the order of the three people matters when determining the possible outcomes. This question can be posed and answered as a whole-class discussion, or in small groups. To conclude and summarize this discussion provide students with the formal definition of **Permutations** and **Combinations**

4. Define a **factorial**, and introduce the following formulas to calculate the number of possible outcomes for Permutations ($P$) and Combinations ($C$).

$$P_{(n,r)} = \frac{n!}{(n-r)!} \quad \text{and} \quad C_{(n,r)} = \frac{n!}{r! (n-r)!}$$

5. Use the following Quiz to help students understand the key differences between permutations and combinations.

**Question 1 of 5**

A book club offers a choice of 8 books from a list of 40, how many ways can a

- A. Permutation
- B. Combination

[Check Answer]
6. Use the following video to illustrate how to calculate the number of outcomes for permutations and combinations using a TI-graphing calculator.

Source: www.youtube.com
Graphing Calculator Tutorial for Permutations and Combinations
7. Conclude this section using examples from the attached sample lesson plan \((at \ the \ right)\) or practice worksheet \((below)\). Have students first determine if the scenario would be labeled as a permutation or combination, then determine the number of outcomes \(using \ the \ formulas \ and \ the \ calculator\).
RESOURCES

SAMPLE UNIT PLANS FOR PROBABILITY

HTTP://WWW.MATHEMATICSVISIONPROJECT.ORG/UPLOADS/1/1/6/3/11636986/SEC2_MOD9_PROB_TN_83113.PDF

JEOPARDY GAME FOR REVIEW
**KEY TERMINOLOGY**

**Combinations** A selection of all or part of a set of objects without regard to the order in which objects are selected (StatTrek, 2014).

**Complementary Events** Two events are described as complementary if they are the only two possible outcomes.

**Conditional Probability** The probability that event A occurs, given event B has occurred. The symbolic notation used to show the conditional probability of event A, given event B, would be $P(A|B)$ (StatTrek, 2014).

**Dependent Events** Events A and B are dependent if the probability of event B happening does depend on whether event A happened.

**Events** An event is any set of outcomes, i.e. a subset of the sample space (Watkins, Schaeffer, & Cobb, 2004).

**Factorial** The product of a whole number and all whole numbers below it until you reach 1.

**Fundamental Principle of Counting** If there are $k$ stages in a process, with $n$ possible outcomes or stage $i$, then the total number of possible outcomes for all $k$ stages taken together is $n_1, n_2, \ldots, n_k$ (Watkins, Schaeffer, & Cobb, 2004).

**Independent Events** Events A and B are independent if the probability of event A happening doesn't depend on whether event B happens.

**Intersection** The intersection of two sets is the set of elements that belong to both sets. The symbolic notation used for the intersection of two sets is $Set_1 \cap Set_2$ (StatTrek, 2014).

**Mutually Exclusive Events** (disjoint events) Events that cannot occur on the same trial.

**Permutations** An arrangement of all or part of a set of objects with regard to order of the arrangement (StatTrek, 2014).

**Probability** A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition) (CCSSI, 2010).

**Probability Model** A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. A uniform probability model assigns equal probability to all outcomes (CCSSI, 2010).

**Sample Space** In a probability model for a random process, a list of the individual outcomes that are to be considered (CCSSI, 2010).

**Tree Diagram** A diagram used for calculating probabilities for sequential events (Watkins, Schaeffer, & Cobb, 2004).

**Union** The union of two sets is the set of elements that belong to one or both sets. The symbolic notion used for the union of two sets is $Set_1 \cup Set_2$ (StatTrek, 2014).


**Fundamental Principle of Counting.** If there are k stages in a process, with n outcomes for stage i, then the total number of possible outcomes for all k stages taken together is $n_1 n_2 \ldots n_k$ (Watkins, Schaeffer, & Cobb, 2004).

**Related Glossary Terms**
Drag related terms here

**Index**
Find Term

**Chapter 2 - Sample Space, tree diagrams and counting events**
**INDEPENDENT EVENTS**

Events A and B are independent if the probability of event A happening doesn’t depend on whether event B happens.

**Related Glossary Terms**

Drag related terms here
MUTUALLY EXCLUSIVE EVENTS

Events that cannot occur on the same trial

Related Glossary Terms
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**Probability**. A number between 0 and 1 used to quantify likelihood for processes with uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition) (CCSSI, 2010).

**Probability model**. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. A uniform probability model assigns equal probability to all outcomes (CCSSI, 2010).

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**Related Glossary Terms**

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SAMPLE SPACE

In a probability model for a random process, a list of the individual outcomes that are considered (CCSSI, 2010).

Events. An event is any set of outcomes, i.e. a subset of the sample space (Watkins, Schaeffer, & Cobb, 2004).

Related Glossary Terms

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TREE DIAGRAMS

A diagram used for calculating probabilities for sequential events (Watkins, Schaeffer, & Cobb, 2004).

Related Glossary Terms

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Related Glossary Terms
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