Quantities for describing the interaction of radiation with matter

Topics covered today

- Kerma
- Absorbed dose
- Comparative examples of energy imparted, energy transferred and net energy transferred
- Exposure, W
- Quantities and units for use in radiation protection

Stochastic Quantities

- Radiant Energy, R
  - R is defined as the energy of particles (excluding rest energy) emitted, transferred or received (ICRU 1980).

Stochastic Quantities

- Energy transferred, ε_{tr} in a volume V

$$\varepsilon_{tr} = (R_{in})_u - (R_{out})_u^{nonr} + \sum Q$$

$(R_{in})_u$ = radiant energy of uncharged particles entering V

$(R_{out})_u^{nonr}$ = radiant energy of uncharged particles leaving V, except that which originated from radiative losses of kinetic energy by charged particles while in V

$\sum Q$ = net energy derived from rest mass in V (m -> E positive, E -> m negative)

Radiative losses = conversion of charged particle energy to photon energy, either bremsstrahlung or in-flight annihilation of positrons.

Kerma

- Definition

$$K = \frac{d(\epsilon_{\mathrm{tr}})_e}{dm} \equiv \frac{d\epsilon_{\mathrm{tr}}}{dm}$$

$(\varepsilon_{tr})_e$ is the expectation value of the energy transferred in the finite volume V during some time interval t

- Nonstochastic quantity only relevant to indirectly ionization radiation
- K depends on the material

Kerma

Kerma is the expectation value of the of the energy transferred to charged particles per unit mass at a point of interest, including radiative-loss energy but excluding energy passed from one charged particle to another.

Kerma can be expressed in terms of ergs/g, rad, or J/kg (Gray)

1 Gy = 1J/kg = 10^2 rad = 10^4 erg/g

Relationship of Kerma to Energy Fluence for Photons

$$K = \Psi \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z}$$

μ_{tr}/ρ is the mass-energy transfer coefficient

Ψ is the energy fluence at a point P

$$K = \int_{E=0}^{E_{max}} \Psi'(E) \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} dE$$

Relationship of Kerma to Energy Fluence for Photons

Average value of (μ_{tr}/ρ) for the spectrum $\Psi'(E)$ is given by

$$\left(\frac{\overline{\mu_{tr}}}{\rho}\right)_{\Psi'(E),Z} = \frac{K}{\Psi} = \frac{\int_E \Psi'(E) \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} dE}{\int_E \Psi'(E)\, dE}$$

Relationship of Kerma to Fluence for Neutrons

Kerma for neutrons could be defined with the same equations as for photons, but it is costumary to describe neutrons fields as fluence fields. Instead of mass-energy transfer coefficients, we use the kerma factor F_n

$$(F_n)_{E,Z} = \left(\frac{\mu_{\text{tr}}}{\rho}\right)_{E,Z} \cdot E$$

Relationship of Kerma to Fluence for Neutrons

$$E\left(\frac{\text{MeV}}{\text{neutron}}\right) \times 1.602 \times 10^{-6} \frac{\text{erg}}{\text{MeV}} \times 10^{-2} \frac{\text{g rad}}{\text{erg}} = E\left(\frac{\text{g rad}}{\text{neutron}}\right)$$

$$K = \Phi \cdot (F_n)_{E,Z} \qquad \text{(rad)}$$

$$K = \int_{E=0}^{E_{\max}} \Phi'(E) \cdot (F_n)_{E,Z}\, dE \qquad \text{(rad)}$$

Relationship of Kerma to Fluence for Neutrons

$$\overline{(F_n)}_{\Phi'(E),Z} = \frac{K}{\Phi} = \frac{\int_E \Phi'(E) \cdot (F_n)_{E,Z}\, dE}{\int_E \Phi'(E)\, dE}$$

Components of Kerma

The kerma for gamma- and x-rays consists of energy transferred to electrons and positrons per unit mass of medium. This kinetic energy can be spent in 2 ways:

1. Coulomb-force interactions with atomic electrons of the absorbing material resulting in the local dissipation of the energy as ionization and excitation in or near the electron track. These are called collisions interactions.

Components of Kerma

2. Radiative interactions with the Coulomb force of atomic nuclei, x-ray photons are emitted as the electron decelerates. The x-ray photons are relatively penetrating compared to electrons and they carry their quantum energy far away from the charged-particle track.

Components of Kerma

In addition, a positron can lose an appreciate portion of its kinetic energy through in-flight annihilation, in which, the kinetic energy possessed by the particle at the instant of annihilation appears an extra quantum energy in the resulting photon.

$$K = K_c + K_r$$

Components of Kerma

$$\epsilon_{tr}^{n} = (R_{in})_u - (R_{out})_u^{nonr} - R'_u + \sum Q = \epsilon_{tr} - R'_u$$

R'_u is the radiant energy emitted as radiative losses by the charged particles which themselves originated in V, regardless of where the radiative losses occur

$$K_c = \frac{d\epsilon_{tr}^{n}}{dm}$$

$$K_c = \Psi \left(\frac{\mu_{en}}{\rho} \right)_{E,Z}$$

Components of Kerma

The collision Kerma (K_c) is the expectation value of the net energy transferred to cp per unit mass at the point of interest, excluding both the radiative-loss energy and energy passed from one cp to another.

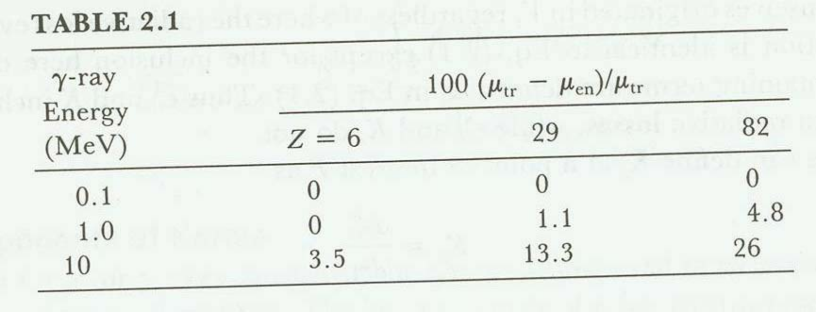


TABLE 2.1

γ-ray Energy (MeV)	$100\,(\mu_{tr} - \mu_{en})/\mu_{tr}$		
	$Z = 6$	29	82
0.1	0	0	0
1.0	0	1.1	4.8
10	3.5	13.3	26

Absorbed Dose

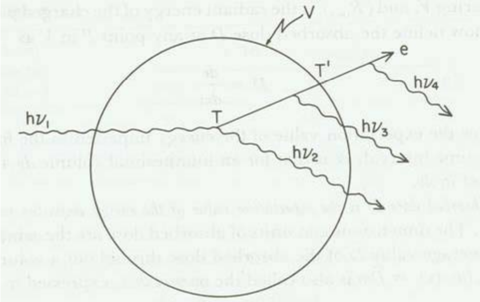
Energy imparted, ε

$$\epsilon = (R_{\text{in}})_u - (R_{\text{out}})_u + (R_{\text{in}})_c - (R_{\text{out}})_c + \Sigma Q$$

$$D = \frac{d\epsilon}{dm}$$

Absorbed dose, D, is the expectation value of the energy imparted to matter per unit mass at a point.
Absorbed dose depends on the material.

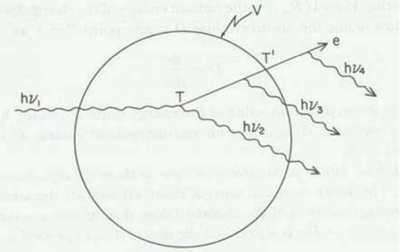
Comparative Examples



Compton interaction followed by bremmssrahlung emission

$$\varepsilon_{tr} = (R_{in})_u - (R_{out})_u^{nonr} + \sum Q$$

Comparative Examples

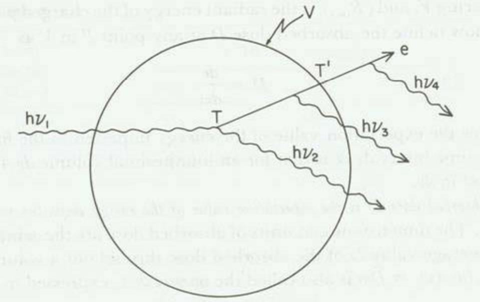


Compton interaction followed by bremmssrahlung emission

$$\varepsilon_{tr} = (R_{in})_u - (R_{out})_u^{nonr} + \sum Q$$

$$\epsilon_{tr} = h\nu_1 - h\nu_2 + 0 = T$$

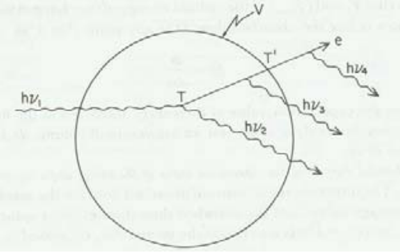
Comparative Examples



Compton interaction followed by bremmssrahlung emission

$$\epsilon_{tr}^{n} = (R_{in})_u - (R_{out})_u^{nonr} - R_u^r + \sum Q = \epsilon_{tr} - R_u^r$$

Comparative Examples

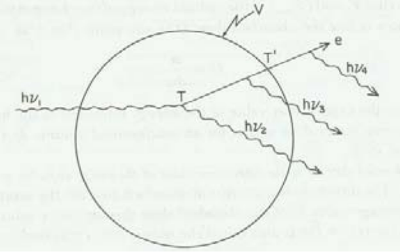


Compton interaction followed by bremmssrahlung emission

$$\epsilon_{\mathrm{tr}}^{n} = (R_{\mathrm{in}})_u - (R_{\mathrm{out}})_u^{\mathrm{nonr}} - R_u^r + \sum Q = \epsilon_{\mathrm{tr}} - R_u^r$$

$$\begin{aligned}\epsilon_{\mathrm{tr}}^{n} &= h\nu_1 - h\nu_2 - (h\nu_3 + h\nu_4) + 0 \\ &= T - (h\nu_3 + h\nu_4)\end{aligned}$$

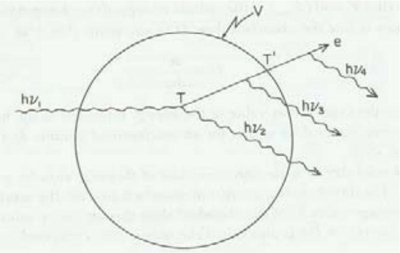
Comparative Examples



Compton interaction followed by bremmssrahlung emission

$$\epsilon = (R_{\mathrm{in}})_u - (R_{\mathrm{out}})_u + (R_{\mathrm{in}})_c - (R_{\mathrm{out}})_c + \sum Q$$

Comparative Examples

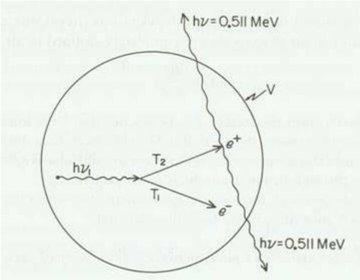


Compton interaction followed by bremmssrahlung emission

$$\epsilon = (R_{\text{in}})_u - (R_{\text{out}})_u + (R_{\text{in}})_c - (R_{\text{out}})_c + \sum Q$$

$$\epsilon = h\nu_1 - (h\nu_2 + h\nu_3 + T') + 0$$

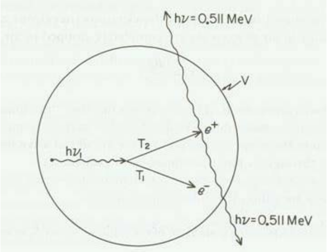
Comparative Examples



Gamma-ray emission pair production and positron annihilation

$$\varepsilon_{tr} = (R_{in})_u - (R_{out})_u^{nonr} + \sum Q$$

Comparative Examples



Gamma-ray emission pair production and positron annihilation

$$\epsilon = \epsilon_{tr} = \epsilon_{tr}^{n} = 0 - 1.022 \text{ MeV} + \sum Q$$

$$\sum Q = h\nu_1 - 2m_0c^2 + 2m_0c^2 = h\nu_1$$

$$\epsilon = \epsilon_{tr} = \epsilon_{tr}^{n} = h\nu_1 - 1.022 \text{ MeV}$$
$$= T_1 + T_2$$

Exposure

- Exposure, X, is by convention defined only for x-ray and gamma-ray photons and air.

$$X = \frac{dQ}{dm}$$

dQ is the absolute value of the total charge of the ions of one sign produced in air when all the electrons (and positrons) liberated by photons in air of mass dm are completely stopped in air.

The ionization arising from the absorption of bremsstrhalung emitted by the electrons is not to be included in dQ.

Exposure

The exposure X is the ionization equivalent of the collision kerma in air, for x- and gamma-rays.

Definition of $\overline{W}$

$\overline{W}$ is the mean energy expended in a gas per ion pair formed.

T_i initial kinetic energy of i^{th} electron set in motion by the x- or gamma-ray in volume dV at point P

g_i the fraction of Ti that is spent by the i^{th} particle along its full path in air in radiative interactions $\sum T_i(1-g_i)$

Definition of $\overline{W}$

$\overline{N_i}$ total number of ion pairs that are produced in air by the i^{th} electron set in motion by the x- or gamma-ray in volume dV at point P

g_i' the fraction of N_i that is spent by the i^{th} particle along its full path in air in radiative interactions

$$\sum N_i(1-g_i')$$

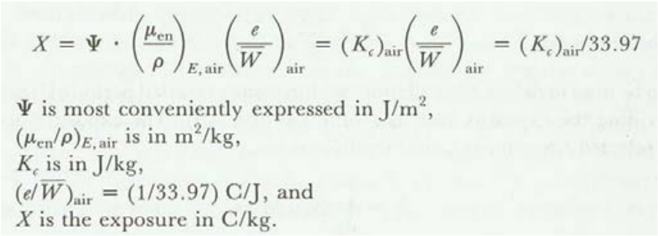
Definition of $\overline{W}$

$$\overline{W} = \frac{\sum T_i(1-g_i)}{\sum N_i(1-g_i')}$$

W = 33.97 eV/ip for dry air

$$\frac{\overline{W}_{\text{air}}}{e} = \frac{33.97\ \text{eV/i.p. (or electron)}}{1.602 \times 10^{-19} C/\text{electron}} \times 1.602 \times 10^{-19}\text{J/eV}$$
$$= 33.97\ \text{J/C}$$

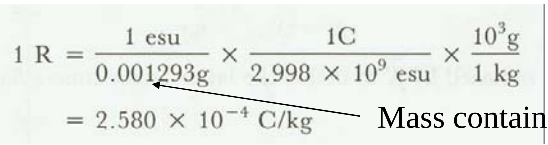
Relation of Exposure to Energy Fluence



$$X = \Psi \cdot \left(\frac{\mu_{en}}{\rho}\right)_{E,\text{air}} \left(\frac{e}{\overline{W}}\right)_{\text{air}} = (K_c)_{\text{air}} \left(\frac{e}{\overline{W}}\right)_{\text{air}} = (K_c)_{\text{air}}/33.97$$

Ψ is most conveniently expressed in J/m^2,
$(\mu_{en}/\rho)_{E,\text{air}}$ is in m^2/kg,
K_c is in J/kg,
$(e/\overline{W})_{\text{air}} = (1/33.97)$ C/J, and
X is the exposure in C/kg.

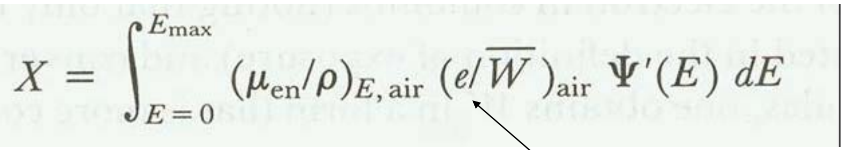
1 R is the exposure that produces, in air, 1 esu of charge
Of either sign per 0.001293 g of air, irradiated by the photons.



$$1\ \text{R} = \frac{1\ \text{esu}}{0.001293\text{g}} \times \frac{1\text{C}}{2.998 \times 10^{9}\ \text{esu}} \times \frac{10^{3}\text{g}}{1\ \text{kg}} = 2.580 \times 10^{-4}\ \text{C/kg}$$

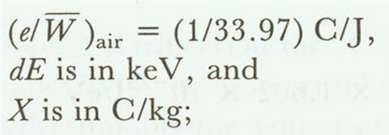
Mass contained in 1 cm^3 at 760 mm Hg and 0^o C

Relation of Exposure to Energy Fluence



$$X = \int_{E=0}^{E_{\max}} (\mu_{en}/\rho)_{E,\text{air}}\ (e/\overline{W})_{\text{air}}\ \Psi'(E)\ dE$$

Exposure spectrum



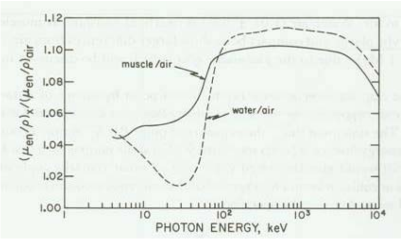
$(e/\overline{W})_{\text{air}} = (1/33.97)$ C/J,
dE is in keV, and
X is in C/kg;

Significance of Exposure

- The energy fluence Ψ is proportional to the exposure X for any given photon energy or spectrum
- The mixture of elements in air is sufficiently similar in “effective atomic number” to that in soft biological tissue to make air an approximately “tissue-equivalent” material with respect to x- or gamma-ray energy absorption. Thus if one is interested in the effects of such radiations in tissue, air may be substituted as a reference medium in a measuring instrument.

Significance of Exposure

- Because of the approximate tissue equivalence of air, the value of kerma in muscle, per unit exposure X, is nearly independent of energy.

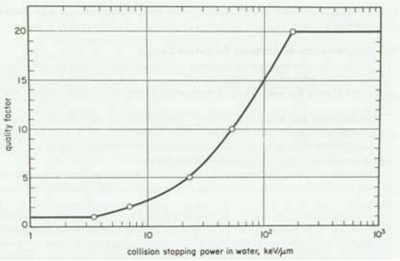


Significance of Exposure

- One can characterize an x-ray field at a point by means of a statement of exposure or exposure rate regardless of whether there is air actually located at the point in question. This means that the photon energy fluence at the point is such that it would give a stated value of X at the point.

Quantities and units for use in radiation protection

- Quality Factor, Q
  - Dimensionless variable weighting factor to be applied to D to provide an estimate of relative human hazard of different types of ionization radiation



Quantities and units for use in radiation protection

- Dose Equivalent, H

$$H \equiv DQN$$

Sievert when D is in Gy and rem when D is in cGy (rad)

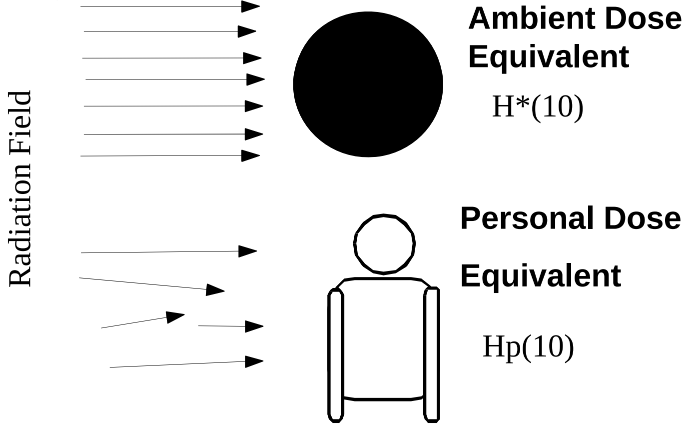
$$H = D\overline{Q}N$$

$$\overline{Q} = \frac{1}{D}\int_0^\infty QD(L_\infty)\,dL_\infty$$

Specification of Ambient Radiation Levels

- Absorbed dose index (D_I)
  - D_I in a field of ionizing radiation is defined as the maximum absorbed dose occurring within a 30 cm diameter sphere of tissue, centered at a point.
- Dose-equivalent index (H_I)
  - H_I is defined as the maximum dose equivalent occurring in the same sphere, also centered at the point.

Operational quantities were defined by ICRU for evaluation of occupational radiation doses to workers and public in general. This is for external sources of radiation only.



Radiation Field

Ambient Dose Equivalent

H*(10)

Personal Dose Equivalent

Hp(10)