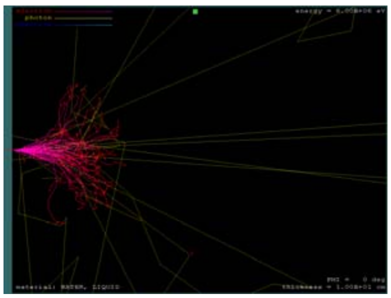
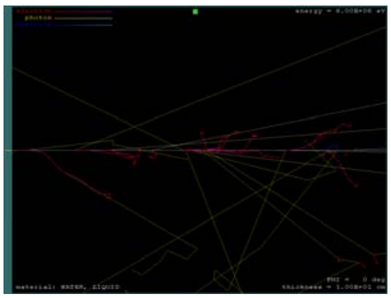
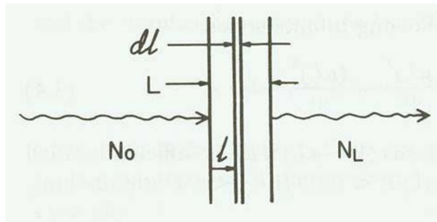
Exponential Attenuation

Electron and photon interactions





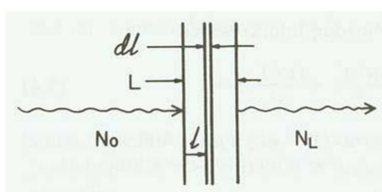
Simple Exponential Attenuation



Each particle either interacts with the flat plate of material producing no secondary radiation or passes straight through the material unchanged in energy or direction.
Let (μ 1) be the probability that an individual particle interacts in a unit thickness of material traversed.

$$dN = -\mu N \, dl$$

Simple Exponential Attenuation



$$\frac{dN}{N} = -\mu \, dl$$

$$\int_{N=N_0}^{N_L} \frac{dN}{N} = -\int_{l=0}^{L} \mu \, dl$$

$$\ln N \Big|_{N_0}^{N_L} = -\mu l \Big|_0^L$$

$$\ln N_L - \ln N_0 = \ln \frac{N_L}{N_0} = -\mu L$$

$$\frac{N_L}{N_0} = e^{-\mu L}$$

μ is the linear attenuation coefficient or just attenuation coefficient or narrow beam attenuation coefficient

Simple Exponential Attenuation

$$\frac{N_L}{N_0} = e^{-\mu L} = 1 - \mu L + \frac{(\mu L)^2}{2!} - \frac{(\mu L)^3}{3!} + \cdots$$

$$\frac{N_L}{N_0} = e^{-\mu L} \cong 1 - \mu L$$

for μ L < 0.05
NL/No is valid within about a tenth of 1 %.

1/μ is the mean free path or relaxation length

Exponential attenuation for plural modes of absorption

$$\mu = \mu_1 + \mu_2 + \cdots$$

Partial linear attenuation coefficient

$$1 = \mu_1/\mu + \mu_2/\mu + \cdots$$

$$\frac{N_L}{N_0} = e^{-(\mu_1 + \mu_2 + \cdots)L}$$

$$\Delta N = N_0 - N_L = N_0 - N_0 e^{-\mu L}$$ All processes

$$\Delta N_x = (N_0 - N_L)\frac{\mu_x}{\mu} = N_0 \left(1 - e^{-\mu L}\right)\frac{\mu_x}{\mu}$$ Single process x

Example 3.1

Example 3.1: Let $\mu_1 = 0.02\ \text{cm}^{-1}$ and $\mu_2 = 0.04\ \text{cm}^{-1}$ be the partial linear attenuation coefficients in the slab shown in Fig. 3.1. Let $L = 5$ cm, and $N_0 = 10^6$ particles. How many particles N_L are transmitted, and how many are absorbed by each process in the slab?

Example 3.1

Example 3.1: Let $\mu_1 = 0.02\ \text{cm}^{-1}$ and $\mu_2 = 0.04\ \text{cm}^{-1}$ be the partial linear attenuation coefficients in the slab shown in Fig. 3.1. Let $L = 5$ cm, and $N_0 = 10^6$ particles. How many particles N_L are transmitted, and how many are absorbed by each process in the slab?

$$N_L = N_0 e^{-(\mu_1 + \mu_2)L} = 10^6\, e^{-(0.02+0.04)5}$$

$$= 7.408 \times 10^5$$

The total number of particles absorbed is

$$N_0 - N_L = (10^6 - 7.408 \times 10^5) = 2.592 \times 10^5$$

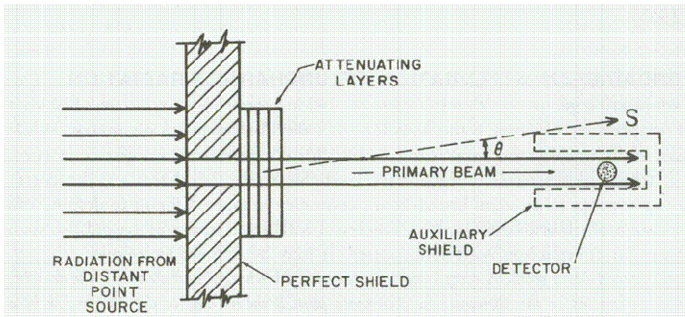
The number absorbed by process 1 is

$$\Delta N_1 = (N_0 - N_L)\frac{\mu_1}{\mu} = 2.592 \times 10^5 \times \frac{0.02}{0.06} = 8.64 \times 10^4$$

and by process 2,

$$\Delta N_2 = (N_0 - N_L)\frac{\mu_2}{\mu} = 2.592 \times 10^5 \times \frac{0.04}{0.06} = 1.728 \times 10^5$$

Narrow Beam Attenuation of Uncharged Radiation



a. *Discrimination* against all scattered and secondary particles that reach the detector, on the basis of particle energy, penetrating ability, direction, coincidence, anticoincidence, time of arrival (for neutrons), etc.

b. *Narrow-beam geometry*, which prevents any scattered or secondary particles from reaching the detector.

Broad-beam attenuation of uncharged radiation

In ideal broad-beam geometry every scattered or secondary uncharged particle strikes the detector, but only if generated in the attenuator by a primary particle on its way to the detector, or by a secondary charged particle resulting from such a primary.

$$\frac{R_L}{R_0} = e^{-\mu_{en}L}$$

In the "straight-ahead approximation" , the μ' is approximated by μ_{en} for thin absorbers layers. In this case, the scattered and secondary particles are supposed to continue straight ahead to until they reach the detector.

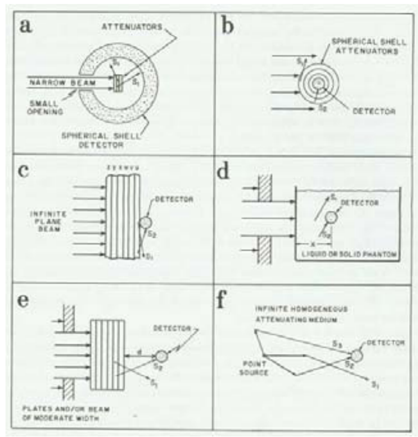
Summary of geometries and attenuation

1. *Narrow-beam geometry.* Only primaries strike the detector; μ is observed for monoenergetic beams.
2. *Narrow-beam attenuation.* Only primaries are counted in N_L by the detector, regardless of whether secondaries strike it; μ is observed for monoenergetic beams.
3. *Broad-beam geometry.* Other than narrow-beam geometry; at least some scattered and secondary radiation strikes the detector.
4. *Broad-beam attenuation.* Scattered and secondary radiation is counted in N_L by the detector. $\mu' < \mu$ is observed. (Note: Narrow-beam attenuation can be obtained in broad-beam geometry if only the primaries are counted in N_L.)

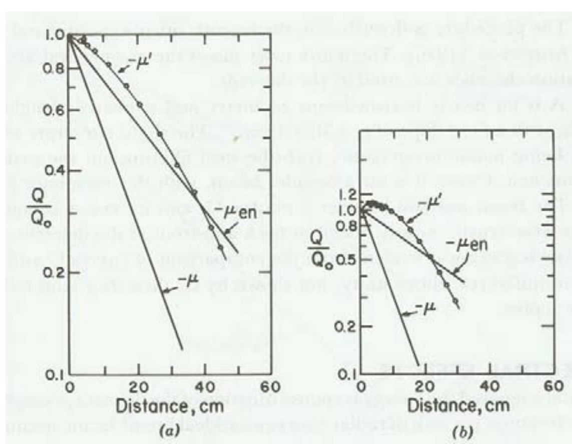
Summary of geometries and attenuation

5. *Ideal broad-beam geometry.* Every scattered or secondary uncharged particle that is generated directly or indirectly by a primary on its way to the detector, strikes the detector. No other scattered or secondary radiation strikes the detector. (Note: Ideal broad-beam geometry can be simulated if each out-scattered particle is replaced by an identical in-scattered particle.)
6. *Ideal broad-beam attenuation.* Ideal broad-beam geometry exists (or is simulated), and the detector responds in proportion to the radiant energy incident on it. In that case $\mu' = \mu_{en}$.

Some Broad Beam Geometries



Some Broad Beam Geometries



^{60}CO (1.25 MeV) *^{203}Hg (0.279 MeV)*

Some Broad Beam Geometries

1. *Excessive in-scattering*, for example caused by backscattering from an attenuating medium located behind the source or the detector, or by crowding of scattered rays in proportion to the cosine of the mean scattering angle θ, so that the particle fluence increases as $1/\cos\theta$ behind a thin plane scatterer in a plane parallel beam (see Fig. 1.5).
2. *Energy-dependence of the detector*, causing it to over-respond to the arriving scattered rays.

Spectral Effects

- So far we have ignored the energy response of the detector, except to require a constant response per unit of radiant energy for ideal broad beam attenuation. For monoenergetic beams this is not a problem.
- In broad beam geometry we have scattered and secondary particles produced that reaches the detector, one must consider the detector response for these particles, which tend to degrade the primaries to lower energies.

Spectral Effects

- The simple counting of particles (fluence) is not useful for attenuation measurements.Our interest in radiation dosimetry is the ability of the radiation to ionize or excite matter. Therefore, it is more important to weight the particles by their energy, in other words, to the energy fluence or the radiant energy arriving at the detector (detector response function).

Spectral Effects

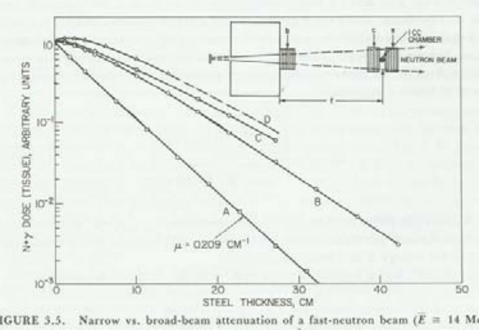


FIGURE 3.5. Narrow vs. broad-beam attenuation of a fast-neutron beam ($\bar{E} = 14$ MeV) in steel. (After Attix et al., 1976.) The detector was a 1-cm^3 ion chamber, located $f = 161$ cm from the shield. The beam at that distance was 3 cm in diameter for curve A, 13 × 13 cm^2 for curve B, and 28 × 28 cm^2 for curves C and D. The attenuators were located at position b for curve A, position c for curves B, C, and D, and also position e for curve D. Curve A is for narrow-beam geometry; curves B, C, and D for progressively broader-beam geometry. Reproduced with permission of *Physics in Medicine and Biology*.

Spectral Effects

- The narrow beam attenuation for a given medium and spectra will have a mean μ that depends on the attenuator thickness L as well as the detector response function.

$$\overline{\mu}_{\Psi,L} = \frac{\int_{E=0}^{E_{\max}} \Psi'_L(E)\ \mu_{E,Z}\ dE}{\int_{E=0}^{E_{\max}} \Psi'_L(E)\ dE}$$

The Buildup Factor

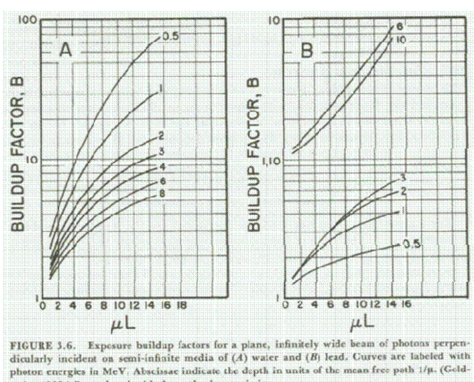


FIGURE 3.6. Exposure buildup factors for a plane, infinitely wide beam of photons perpendicularly incident on semi-infinite media of (A) water and (B) lead. Curves are labeled with photon energies in MeV. Abscissae indicate the depth in units of the mean free path 1/μ. (Goldstein, 1957.) Reproduced with the author's permission.

$$B = \frac{\text{quantity due to primary} + \text{scattered and secondary radiation}}{\text{quantity due to primary radiation alone}}$$

$$\frac{\Psi_L}{\Psi_0} = Be^{-\mu L}$$

$$\frac{\Psi_L}{\Psi_0} = Be^{-\mu L} \equiv e^{-\overline{\mu}' L}$$

$$\overline{\mu}' \equiv \mu - \frac{\ln B}{L}$$

The Buildup Factor

TABLE 3.1. Comparison of Exposure Buildup Factor B and Mean Effective Attenuation Coefficient $\bar{\mu}'$ for a Plane Beam of 1-MeV γ-Rays in Water[a]

B	μL	$X_L/X_o = Be^{-\mu L}$	L (cm)	$\bar{\mu}'$ (cm^{-1})	$\bar{\mu}'/\mu$	$\bar{\mu}'/\mu_{en}$
3	1.7	0.548	24	0.025	0.35	0.81
6	4.0	0.110	57	0.039	0.55	1.26
10	6.3	1.84×10^{-2}	89	0.045	0.64	1.46
20	10.9	3.69×10^{-4}	154	0.051	0.72	1.65
30	14.6	1.37×10^{-5}	207	0.054	0.76	1.75

[a] $\mu = 0.0706\ cm^{-1}$; $\mu_{en} = 0.0309\ cm^{-1}$.

The Buildup Factor

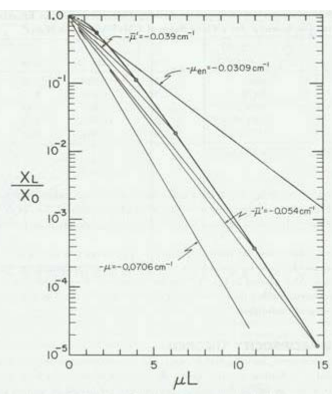


FIGURE 3.7. Graph of data in Table 3.1, for a broad plane beam of 1-MeV γ-rays in water. Note that any slope in this graph is equal to $[\ln (X_L/X_0)/[L(cm)]$, even though the abscissa is labelled μL, that is, the depth L in units of mean free path $1/\mu$.

The reciprocity theorem

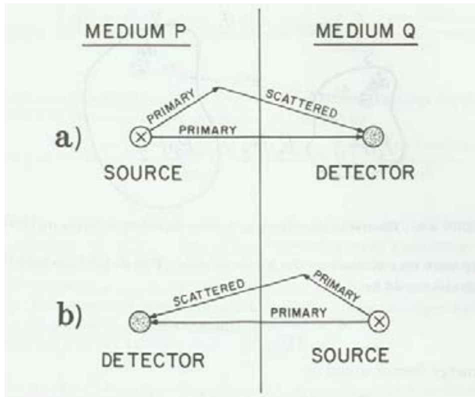


FIGURE 3.8. The reciprocity theorem in radiation transport (see text).

Exact for P=Q
For P≠Q
Exact only if scattering is the same or approximately the same as in the case of Compton effect

The reciprocity theorem

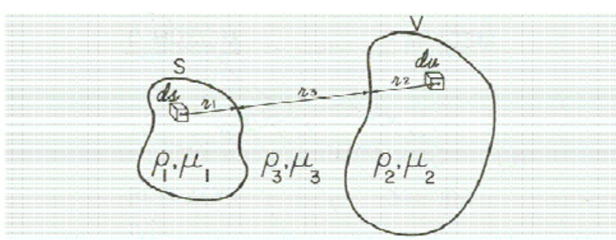


FIGURE 3.9. Illustration of reciprocity theorem according to Mayneord (1945).

The integral dose* in a volume V due to a γ-ray source uniformly distributed throughout source volume S is equal to the integral dose that would occur in S if the same activity density per unit mass were distributed throughout V.

Proof of the reciprocity theorem

Homework assignment for Monday

Medium: Air

If there were no attenuation, the photon fluence Φ at dv for an irradiation time of Δt

$$\Phi = \frac{dA\ \Delta t}{4\pi r^2} \quad \text{(photons/m}^2\text{)}$$

$$\Psi = 1.602 \times 10^{-13}\ \Phi E \quad (\text{J/m}^2)$$

$$K_c = \Psi \left(\frac{\mu_{en}}{\rho_2}\right)_{E,V} \quad (\text{J/kg})$$

$dA = A'\rho_1 ds$
A'= specific activity (Bq/kg)
ds = elementary volume (m3)
Assumption:
decay of one photon with energy E per disentegration

Kerma in volume V due to activity in volume S
Also, $D=K_c$

The reciprocity theorem

$$D = 1.602 \times 10^{-13} \frac{A'\rho_1\ \Delta t\ E(\mu_{en}/\rho_2)_{E,V}\ ds}{4\pi r^2}$$

Still considering no attenuation

Only primary radiation → $e^{(-\mu_1 r_1 - \mu_2 r_2 - \mu_3 r_3)}$. Dose at dv due to S

$$D_{tot} = \frac{1.602 \times 10^{-13} A'\rho_1\ \Delta t\ E(\mu_{en}/\rho_2)_{E,V}}{4\pi} \int_S \frac{e^{-(r_1\mu_1 + r_2\mu_2 + r_3\mu_3)}}{r^2}\, ds$$

$D(V,S)$ = integral of $D_{tot}\rho_2 dv$

$$D(V,S) = \frac{1.602 \times 10^{-13} A'\rho_1\rho_2\ \Delta t\ E(\mu_{en}/\rho_2)_{E,V}}{4\pi} \int_V\int_S \frac{e^{-(r_1\mu_1 + r_2\mu_2 + r_3\mu_3)}}{r^2}\, dv\, ds$$

$$D(S,V) = \frac{1.602 \times 10^{-13} A'\rho_1\rho_2\ \Delta t\ E(\mu_{en}/\rho_1)_{E,S}}{4\pi} \int_S\int_V \frac{e^{-(r_1\mu_1 + r_2\mu_2 + r_3\mu_3)}}{r^2}\, ds\, dv$$

$D(V,S)=D(S,V)$ if $(\mu_{en}/\rho_2)_{E,V} = (\mu_{en}/\rho_1)_{E,S}$

The reciprocity theorem

If *S* and *V* in Fig. 3.9 contain identical, uniformly distributed total activities, they will each deliver to the other the same average absorbed dose.

If all the activity in *S* is concentrated at an internal point *P*, then the dose at *P* due to the distributed source in *V* equals the average dose in *V* resulting from an equal source at *P*.

The dose at any internal point P in S due to a uniformly distributed source throughout S itself is equal to the average absorbed dose in S resulting from the same total source concentrated at P.