Radioactive Decay

Total radioactive decay constant

$$-\frac{dN}{dt} = \lambda N$$

λ is the total radioactive decay constant.
N is the total no. of identical radioactive atoms
λN is the activity
It is a constant proportional to the rate of change of number of atoms as a function of time.
1/λ is the probability that an atom with decay during a time interval.

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_{0}^{t} \lambda dt$$

Partial decay constants

- If nucleus has more than one mode of disintegration (different daughter products)

$$\lambda = \lambda_A + \lambda_B + \lambda_C + \ldots$$

- The total activity is

$$N\lambda = N\lambda_A + N\lambda_B + N\lambda_C + \ldots$$

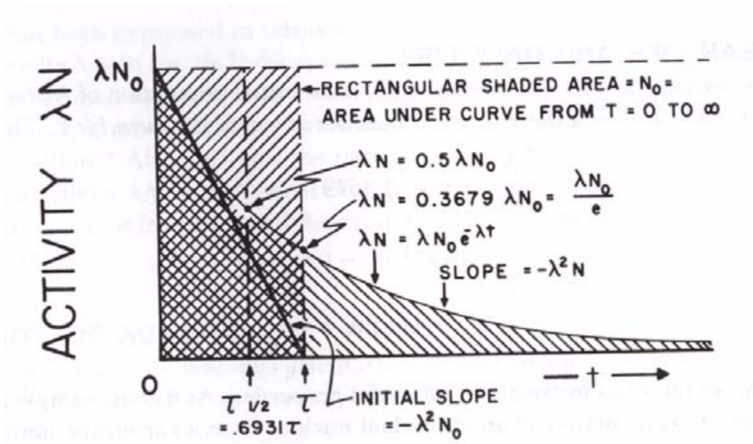
- The partial activity is

$$N_i\lambda = N_0\lambda_i e^{-\lambda t}$$

Units of Activity

- Curie (Ci) = no. of dis/s occurring in a mass of 1 g of ^{226}Ra
- Now 1 Ci = 3.7 x 10^{10} dis/s = 0.988 mg ^{226}Ra
- New SI units → 1 Bq = 1 dis/s

Mean Life and Half Life



Radioactive Parent-Daughter Relationships

$$\frac{dN_2}{dt} = \lambda_{1A} N_1 - \lambda_2 N_2$$

$$= \lambda_{1A} (N_1)_0 e^{-\lambda_1 t} - \lambda_2 N_2 \qquad 6.14$$

$$N_2 = (N_1)_0 \left(x_1 e^{-\lambda_1 t} + x_2 e^{-\lambda_2 t}\right) \qquad 6.15$$

Differentiate 6.15 with respect to t substitute 6.15 and 6.14

$$x_1 = \frac{\lambda_{1A}}{\lambda_2 - \lambda_1} \qquad x_2 = -x_1 = \frac{-\lambda_{1A}}{\lambda_2 - \lambda_1}$$

$N_2 = 0$ when $t=0$

Radioactive Parent-Daughter Relationships

N_2=0 at t=0 and substituting x_1 and x_2

$$\lambda_2 N_2 = \lambda_{1A}(N_1)_0 \frac{\lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

$$\lambda_1 N_1 = \lambda_1 (N_1)_0 e^{-\lambda_1 t}$$

$$\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_{1A}}{\lambda_1} \cdot \frac{\lambda_2}{\lambda_2 - \lambda_1}(1 - e^{-(\lambda_2 - \lambda_1)t})$$

For only one daughter product $\lambda_{1A} = \lambda_1$

$$\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_2 - \lambda_1}(1 - e^{-(\lambda_2 - \lambda_1)t})$$

Equilibria in Parent-Daughter Activities

$$\frac{d(\lambda_2 N_2)}{dt} = 0 = (-\lambda_1 e^{-\lambda_1 t_m} + \lambda_2 e^{-\lambda_2 t_m})$$

Find maximum

$$\lambda_1 e^{-\lambda_1 t_m} = \lambda_2 e^{-\lambda_2 t_m}$$

$$\frac{\lambda_2}{\lambda_1} = e^{(\lambda_2 - \lambda_1)t_m}$$

$$\ln \frac{\lambda_2}{\lambda_1} = (\lambda_2 - \lambda_1)\, t_m$$

$$t_m = \frac{\ln(\lambda_2/\lambda_1)}{\lambda_2 - \lambda_1}$$

Time for maximum Activity

Daughter longer-lived than parent

$$\lambda_2 < \lambda_1$$

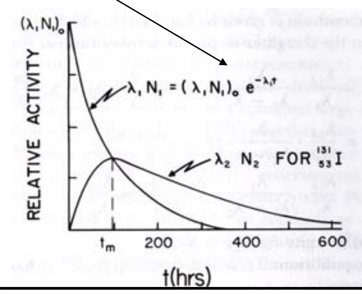
$$\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_{1A}}{\lambda_1} \cdot \frac{\lambda_2}{\lambda_2 - \lambda_1}(1 - e^{-(\lambda_2 - \lambda_1)t}) \longrightarrow \frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_{1A}}{\lambda_1} \cdot \frac{\lambda_2}{\lambda_1 - \lambda_2}(e^{(\lambda_1 - \lambda_2)t} - 1)$$

$$\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_2 - \lambda_1}(1 - e^{-(\lambda_2 - \lambda_1)t}) \longrightarrow \frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_1 - \lambda_2}(e^{(\lambda_1 - \lambda_2)t} - 1)$$

$$^{131m}_{52}\mathrm{Te} \xrightarrow[\tau_{1/2} = 30\,\mathrm{h}]{\beta^-} {}^{131}_{53}\mathrm{I} \xrightarrow[\tau_{1/2} = 193\,\mathrm{h}]{\beta^-} {}^{131}_{54}\mathrm{Xe}$$

$$\lambda_1 = 2.31 \times 10^{-2}\ \mathrm{h}^{-1}, \qquad \lambda_2 = 3.59 \times 10^{-3}\ \mathrm{h}^{-1}$$

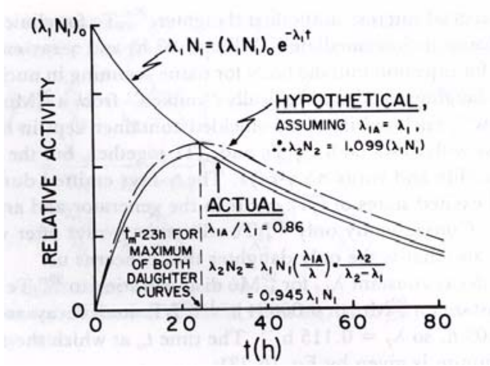
$$\therefore\ \lambda_1 > \lambda_2$$



Daughter shorter-lived than parent

$$\lambda_2 > \lambda_1$$

$$\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_{1A}}{\lambda_1} \cdot \frac{\lambda_2}{\lambda_2 - \lambda_1}(1 - e^{-(\lambda_2 - \lambda_1)t}) \xrightarrow{t \gg t_m} \frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_{1A}}{\lambda_1} \cdot \frac{\lambda_2}{\lambda_2 - \lambda_1}$$



transient equilibrium

$^{99}_{42}$Mo decays to $^{99m}_{42}$Tc decays to $^{99}_{43}$Tc 86% of the time

$^{99}_{42}$Mo decays to $^{99m}_{43}$Tc 14% of the time

$T_{1/2}$ = 66.7 hr for parent
$T_{1/2}$ = 6.03 h for daughter

Only daughter much shorter-lived than parent

$$\lambda_2 >> \lambda_1$$

$$\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_2 - \lambda_1} \cong 1$$

Secular equilibrium

$$^{226}_{88}\text{Ra} \xrightarrow[\substack{\tau_{1/2} = 1602\text{y} \\ \lambda_1 = 1.1845 \times 10^{-6}\text{d}^{-1}}]{\alpha, \gamma} {}^{222}_{86}\text{Rn} \xrightarrow[\substack{\tau_{1/2} = 3.824\text{d} \\ \lambda_2 = 0.18125\text{d}^{-1}}]{\alpha} {}^{218}_{84}\text{Po}$$

Removal of daughter products

- In diagnostics or therapeutic applications of short lived radioisotopes (e.g. ^{99m}Tc), it is useful to remove the daughter product at the time of its maximum activity.

Activity at time t=t_2

$$\lambda_2 N_2 = \lambda_{1A} (N_1)_0 \, e^{-\lambda_1 t_1} \frac{\lambda_2}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 (t_2 - t_1)} - e^{-\lambda_2 (t_2 - t_1)}\right)$$

Initial activity of parent source at time t=0

A_{th} daughter is completed removed at time t=t_1

Radioactivation by nuclear interactions

Number of target atoms ⟶ $$N_t = \frac{N_A m}{A}, \qquad (6.38)$$

where N_A = Avogadro's constant (atoms/mole),
A = gram-atomic weight (g/mole), and
m = mass (g) of *target atoms only* in the sample. (This is equal to the product of the gross sample mass by the weight fraction of target atoms present in the sample.)

$$\left(\frac{dN_{\text{act}}}{dt}\right)_0 = \varphi N_t \sigma$$

$$\left(\frac{d(\lambda N_{\text{act}})}{dt}\right)_0 = \lambda \varphi N_t \sigma \qquad (\text{Bq s}^{-1})$$

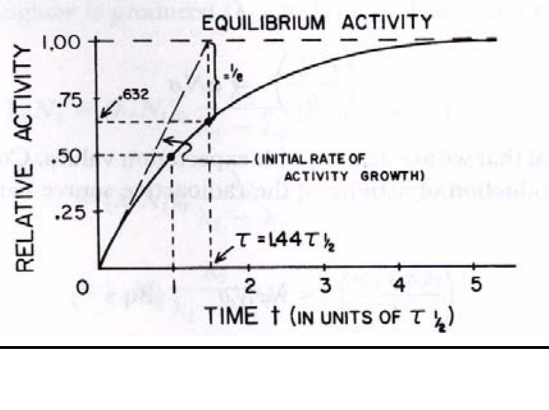
$$\frac{dN_{\text{act}}}{dt} = \varphi N_t \sigma - \lambda N_{\text{act}}$$

Radioactivation by nuclear interactions

$$(\lambda N_{\text{act}})_e = \varphi N_t \sigma \qquad (\text{Bq})$$

$$\lambda N_{\text{act}} = 0 \text{ at } t = 0$$

$$\lambda N_{\text{act}} = (\lambda N_{\text{act}})_e (1 - e^{-\lambda t}) = \varphi N_t \sigma (1 - e^{-\lambda t})$$



Exposure rate constant

$$\Gamma_\delta = \frac{l^2}{A}\left(\frac{dX}{dt}\right)_\delta$$

Includes characteristics x-rays and bremsstrahlung

Specific gamma ray constant Γ only accounts for gamma-rays

TABLE 6.1. Data for Selected γ-Ray Sources[a]

Radionuclide	Half-Life	γ-Photon Energy (MeV)	Specific γ-Ray Constant[b] (R cm^2 mCi^{-1} h^{-1})	Exposure-Rate Constant[b] (R cm^2 mCi^{-1} h^{-1})
^{137}Cs	30.0 y	0.6616	3.200	3.249
^{51}Cr	27.72 d	0.3200	0.1827	0.1827
^{60}Co	5.26 y	1.173–1.322[c]	12.97	12.97
^{198}Au	2.698 d	0.4118–1.088[c]	2.309	2.357
^{125}I	60.25 d	0.03548	0.04194	1.315
^{192}Ir	74.2 d	0.1363–1.062[c]	3.917	3.970
^{226}Ra[d]	1602 y	0.0465–2.440[c]	8.996[e]	10.07
^{182}Ta	115.0 d	0.0427–1.453[c]	7.631	7.753

[a] NCRP (1974).

[b] The specific γ-ray constants and exposure-rate constants were calculated by L. T. Dillman from decay-scheme data, assuming $\overline{W}_{air}$ = 33.70 eV/i.p. Values in the present table have been adjusted downward to be consistent with $\overline{W}_{air}$ = 33.97 eV/i.p. Contributions to these constants by photons below 11.3 keV were excluded.

[c] Minimum and maximum values included in the calculation of specific γ-ray constant and exposure-rate constant.

[d] With daughters.

[e] This value differs from the currently accepted value of 8.35 R cm^2 mCi^{-1} h^{-1} for radium because the value 8.996 was calculated for no filtration. The value of 8.35 is for a filter of 0.5-mm platinum and includes such secondary radiations as may be generated in the platinum filter; it corresponds to 8.25 R cm^2 mg^{-1} h^{-1}, since 1 mg = 0.988 mCi.