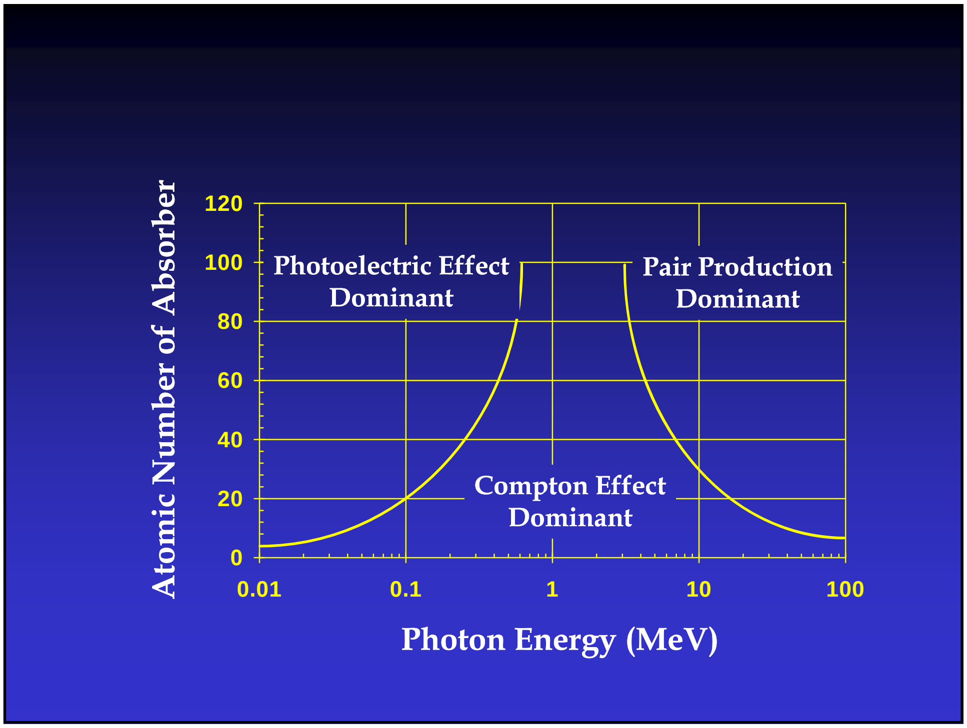
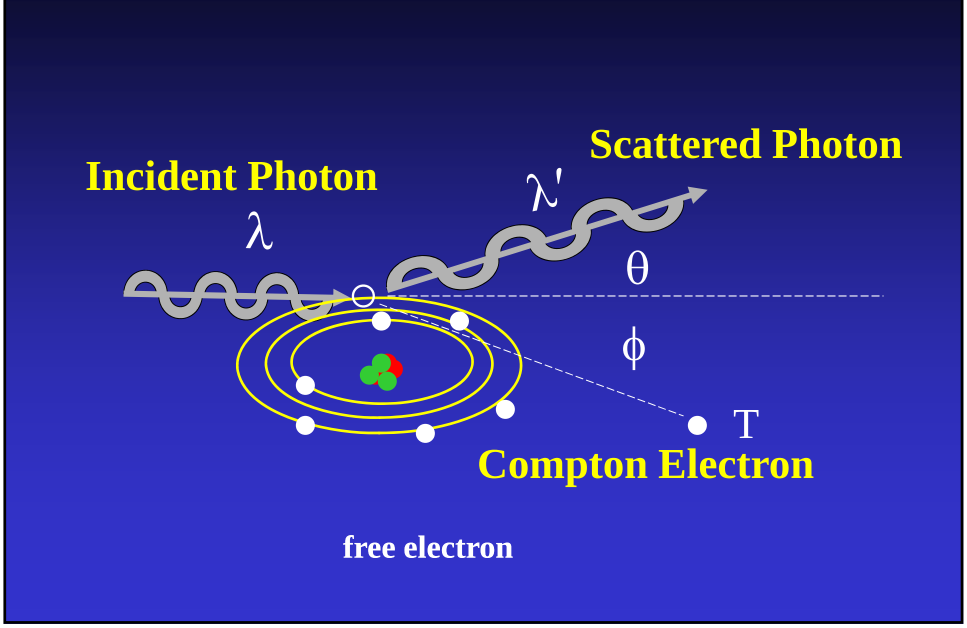
Interaction of Gamma- and X-ray in Matter

Five Interactions

- Compton effect
- Photoelectric effect
- Pair production
- Rayleigh (coherent) scattering
- Photonuclear interactions



Compton Effect



Kinematics

These equations are results of applying conservation of energy and momentum to the Compton interaction

$$h\nu' = \frac{h\nu}{1 + (h\nu/m_0c^2)(1 - \cos\varphi)}$$

$$T = h\nu - h\nu'$$

$$\cot\theta = \left(1 + \frac{h\nu}{m_0c^2}\right)\tan\left(\frac{\varphi}{2}\right)$$

$$h\nu' = \frac{h\nu}{1 + (h\nu/m_0c^2)(1 - \cos\varphi)}$$

$$T = h\nu - h\nu'$$

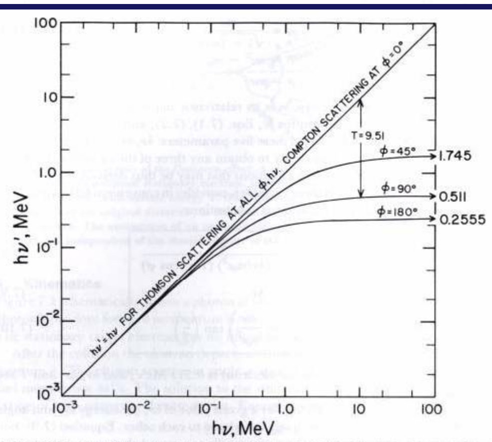


FIGURE 7.3. Graphical representation of the kinematic relationship of $h\nu$, $h\nu'$, and T in the Compton effect, as described by Eqs. (7.8) and (7.9). Curves are shown only for $\varphi = 0$, 45°, 90° and 180°. Note that T is to be interpreted as the vertical separation of any φ-curve from the $\varphi = 0$ diagonal. In the case shown ($h\nu = 10$ MeV, $\varphi = 90°$), $T = 9.51$ MeV.

$$\cot\theta = \left(1 + \frac{h\nu}{m_0c^2}\right)\tan\left(\frac{\varphi}{2}\right)$$

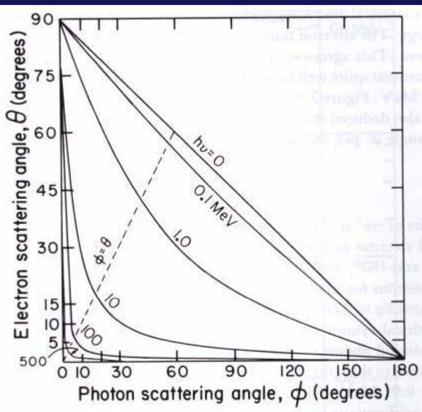


FIGURE 7.4. Relationship of the electron scattering angle θ to the photon scattering angle φ in the Compton effect, from Eq. (7.10). Curves are shown for the incident photon energies 0, 0.1, 1.0, 10, 100, and 500 MeV. The dashed line is the locus where θ = φ, when the electron and photon are scattered at equal angles on opposite sides of the incident photon's direction.

Interaction cross section

Thomson Scattering

classical electron radius

$$\frac{d_e\sigma_0}{d\Omega_\varphi} = \frac{r_0^2}{2}(1 + \cos^2\varphi)$$

hν=hν'

Valid only for non-relativistic approximation

Klein-Nishina cross sections for photon scattering at angle φ

$$\frac{d_e\sigma}{d\Omega_\varphi} = \frac{r_0^2}{2}\left(\frac{h\nu'}{h\nu}\right)^2\left(\frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} - \sin^2\varphi\right)$$

Use of Dirac's relativistic theory of the electron

$$\frac{d_e\sigma}{d\Omega_\varphi} = \frac{r_0^2}{2}\left(\frac{h\nu'}{h\nu}\right)^2\left(\frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} - \sin^2\varphi\right)$$

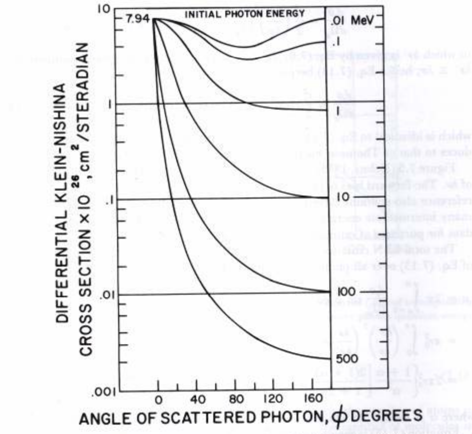


FIGURE 7.5. Differential Klein–Nishina cross section, $d_e\sigma/d\Omega_\varphi$ [see Eq. (7.13)] vs. angle φ of the scattered photon, for $h\nu$ = 0.01, 0.1, 1.0, 10, 100, and 500 MeV. This shows the angular distribution, per unit solid angle, of the scattered photons resulting from the Compton effect. (After Nelms, 1953.)

$$_e\sigma = 2\pi \int_{\varphi=0}^{\pi} \frac{d_e\sigma}{d\Omega_\varphi} \sin\varphi \, d\varphi$$

$$= \pi r_0^2 \int_0^{\pi} \left(\frac{h\nu'}{h\nu}\right)^2 \left(\frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} - \sin^2\varphi\right) \sin\varphi \, d\varphi$$

$$= 2\pi r_0^2 \left\{ \frac{1+\alpha}{\alpha^2}\left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha}\right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}$$

$$\alpha = \frac{h\nu}{m_0 c^2}$$

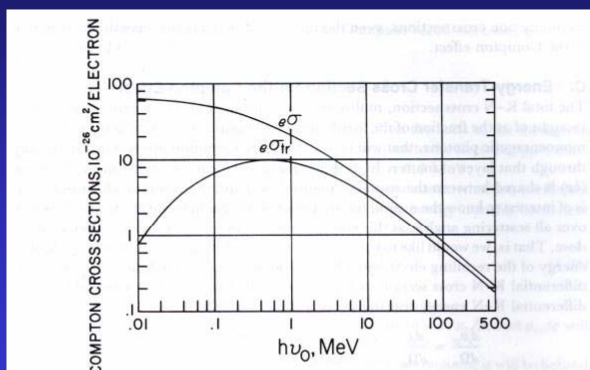


FIGURE 7.6. Klein–Nishina (Compton-effect) cross section per electron ($_e\sigma$) and corresponding energy-transfer cross section per electron ($_e\sigma_{tr}$) as a function of primary photon quantum energy $h\nu$. (After Nelms, 1953.)

$${}_e\sigma \propto Z^0$$

$${}_a\sigma = Z \cdot {}_e\sigma \qquad (\text{cm}^2/\text{atom})$$

Avogrado’s number

$$\frac{\sigma}{\rho} = \frac{N_A Z}{A}\,{}_e\sigma \qquad (\text{cm}^2/\text{g})$$

Number of grams per mole of material

Energy transfer cross section

Total K-N cross section, multiplied by a unit thickness of 1 e/cm^2 may be thought as the fraction of the incident energy fluence, carried by a beam of many monoenergetic photons

What is the overall fraction of hν that is given to the electrons, averaged over all scattering angles? This energy contributes to the kerma and therefore to dose.

hν -hν’

$$\frac{d_e\sigma_{\text{tr}}}{d\Omega_\varphi} = \frac{d_e\sigma}{d\Omega_\varphi} \cdot \frac{T}{h\nu}$$

$$\frac{\overline{T}}{h\nu} = \frac{{}_e\sigma_{tr}}{{}_e\sigma}$$

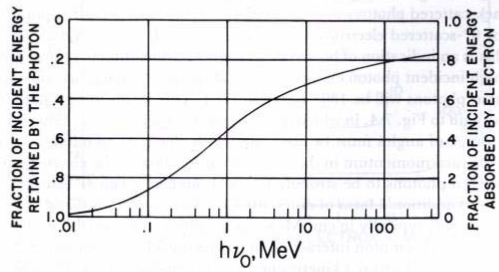


FIGURE 7.7. Mean fraction ($\overline{T}/h\nu$) of the incident photon's energy given to the recoiling electron in Compton interactions, averaged over all angles (right ordinate). Also, mean fraction ($h\bar{\nu}'/h\nu$) of energy retained by the scattered photon (left ordinate).

Differential K-N cross section for electron scattering at angle θ, per unit solid angle per electron

$$\frac{d_e\sigma}{d\Omega_\theta} = \frac{d_e\sigma}{d\Omega_\varphi} \cdot \frac{(1+\alpha)^2(1-\cos\varphi)^2}{\cos^3\theta}$$

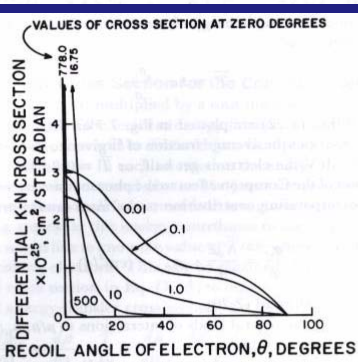


FIGURE 7.8. Differential Klein–Nishina cross section $d_e\sigma/d\Omega_\theta$ vs angle θ of the scattered electron for $h\nu$ = 0.01, 0.1, 1, 10, and 500 MeV. This shows the angular distribution, per unit solid angle, of the recoil electrons resulting from the Compton effect. (After Nelms, 1953.)

Energy distribution of electrons, averaged over all scattering angles

$$\frac{d_e\sigma}{dT} = \frac{\pi r_0^2 m_0 c^2}{(h\nu')^2} \times \left\{ \left[\frac{m_0 c^2 T}{(h\nu)^2}\right]^2 + 2\left[\frac{h\nu'}{h\nu}\right]^2 + \frac{h\nu'}{(h\nu)^3}\left[(T - m_0c^2)^2 - (m_0c^2)^2\right] \right\}$$

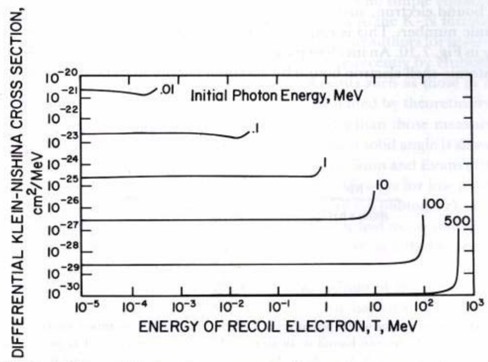
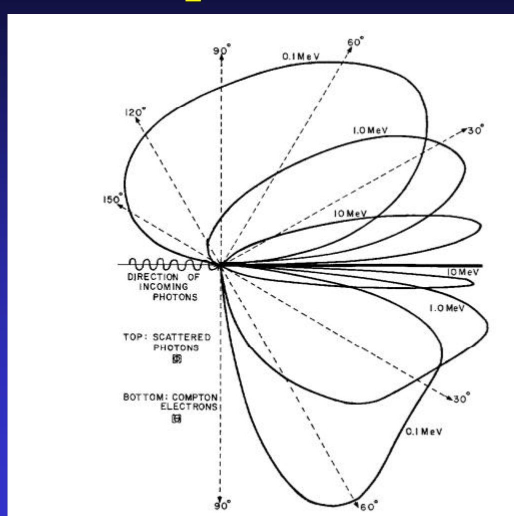


FIGURE 7.9. Differential Klein–Nishina cross section $d_e\sigma/dT$ expressing the initial energy spectrum of Compton recoiling electrons. (After Nelms, 1953.)

Compton Effect

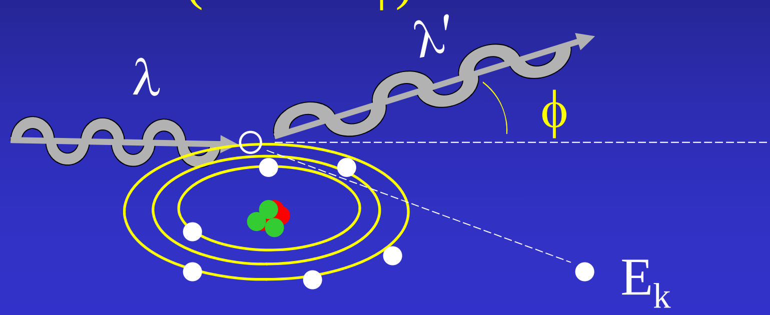


Compton Effect

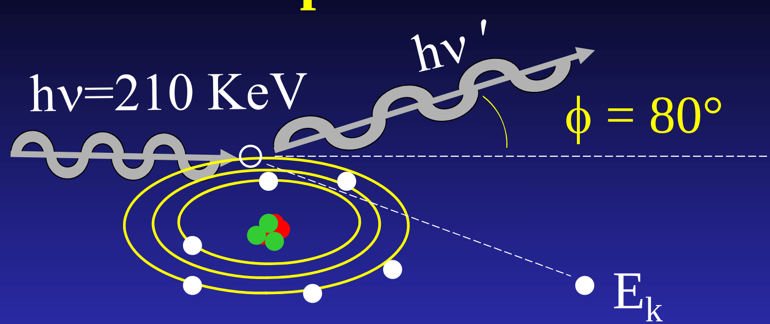
$h\nu(\mathrm{KeV}) = 12.4/\ \lambda(\text{Å})$

$\lambda' = \lambda + \Delta\lambda$

$\Delta\lambda = 0.0243(1 - \cos\phi)$



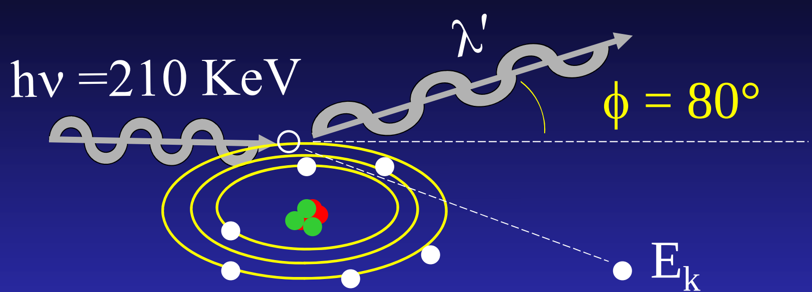
Compton Effect



What is $h\nu'$?

What is E_k ?

Compton Effect

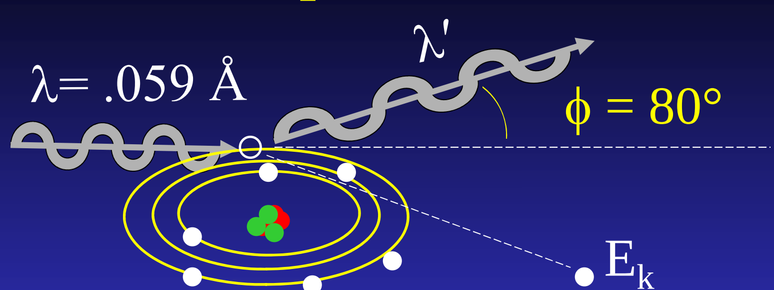


$h\nu = 12.4/\ \lambda(Å)$

$\lambda = 12.4/210 = .059Å$

$\Delta\lambda = 0.0243(1 - \cos 80°) = 0.020Å$

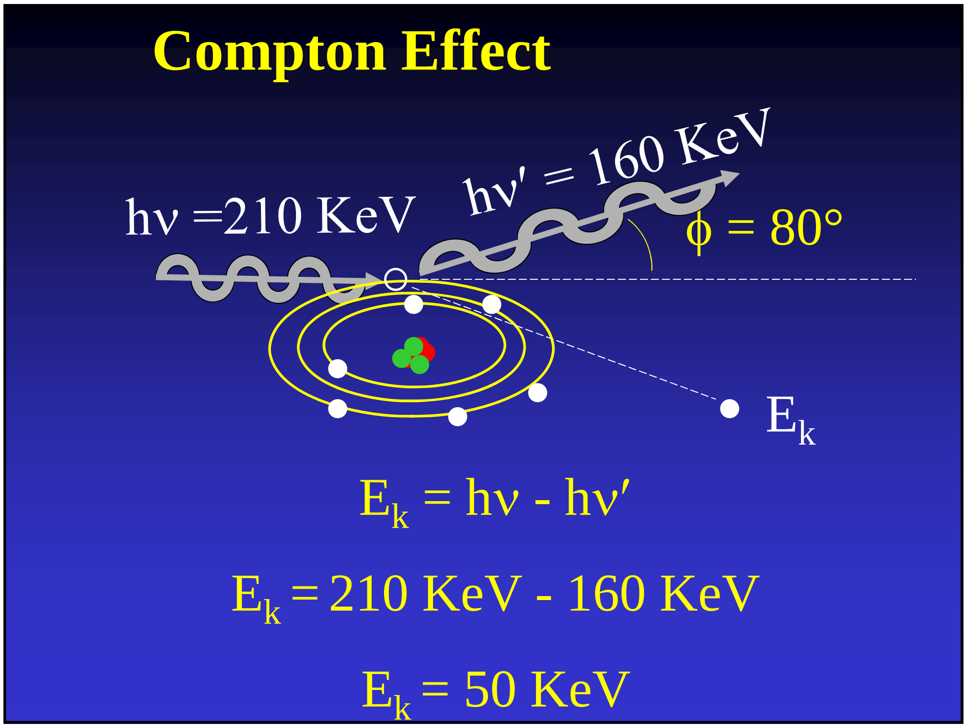
Compton Effect

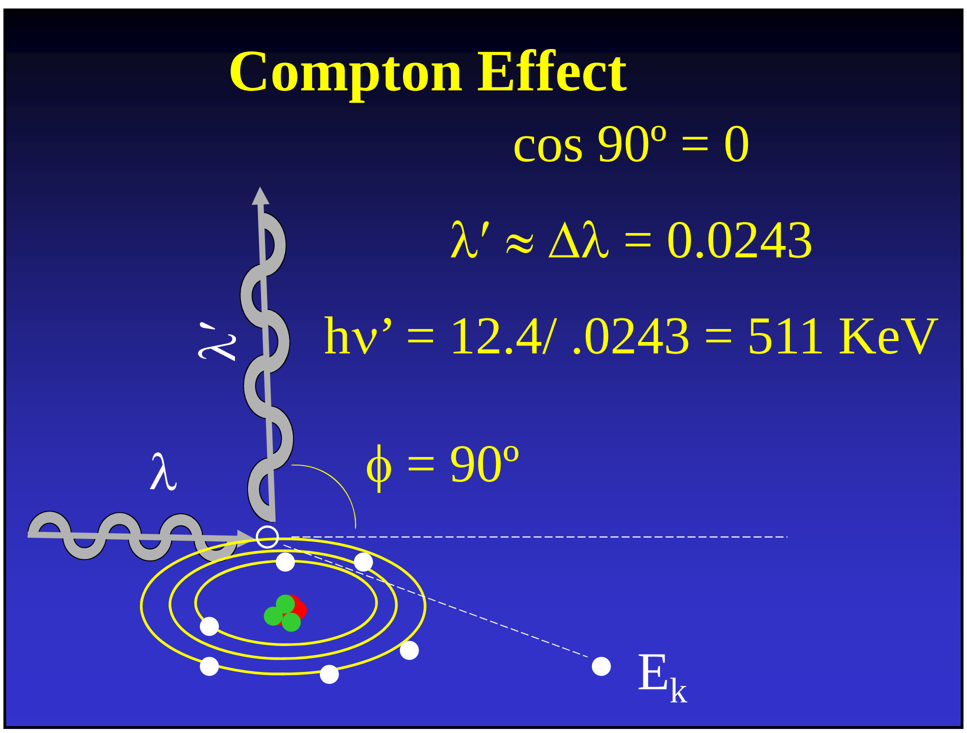


$\lambda' = \lambda + \Delta\lambda = .059Å\ + 0.020Å = .079Å$

$h\nu'(KeV) = 12.4/\ \lambda'\ (Å)$

$12.4/.079 = 160$ KeV


Compton Effect
hν =210 KeV
hν′ = 160 KeV
ϕ = 80°
Ek
Ek = hν - hν′
Ek = 210 KeV - 160 KeV
Ek = 50 KeV

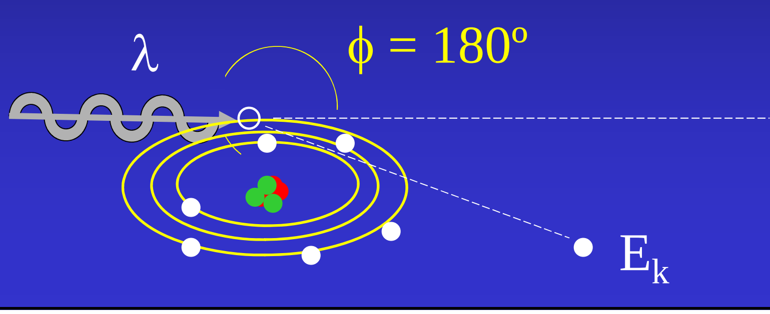

Compton Effect
cos 90º = 0
λ′ ≈ Δλ = 0.0243
hν’ = 12.4/ .0243 = 511 KeV
λ′
λ
ϕ = 90º
Ek

Compton Effect

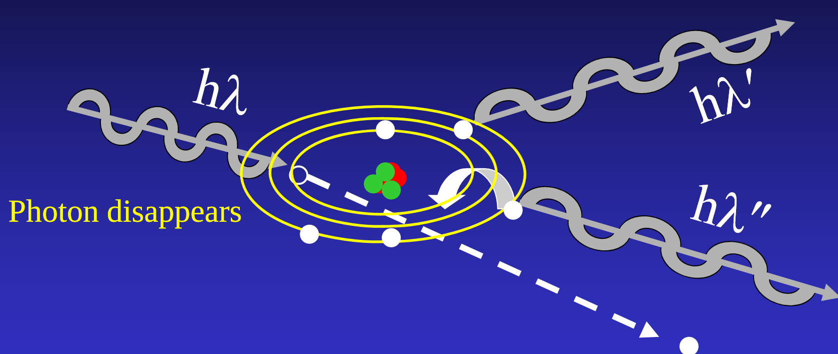
$\cos 180^o = -1$

$\lambda' \approx \Delta\lambda = 0.0243 \cdot 2$

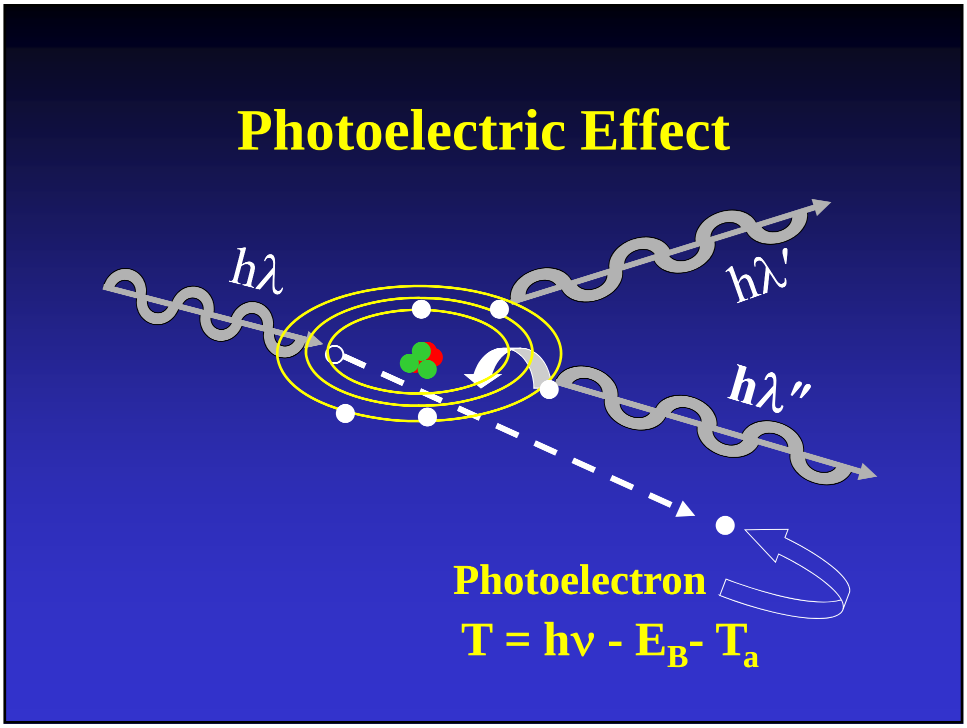
$h\nu' = 12.4 / .0486 = 255$ KeV

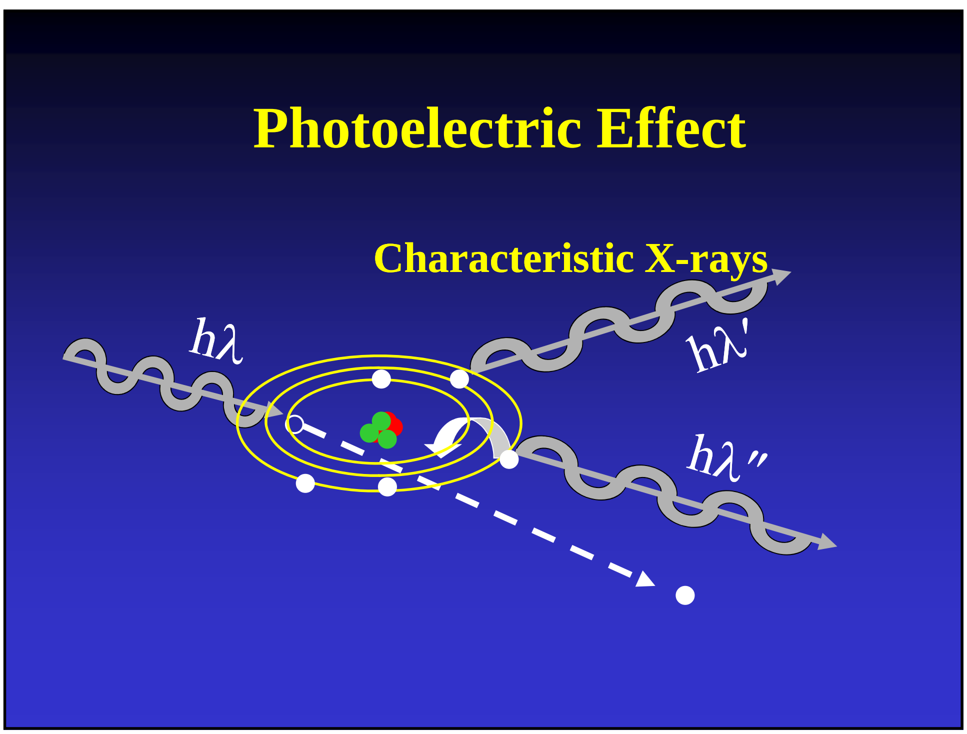


Photoelectric Effect



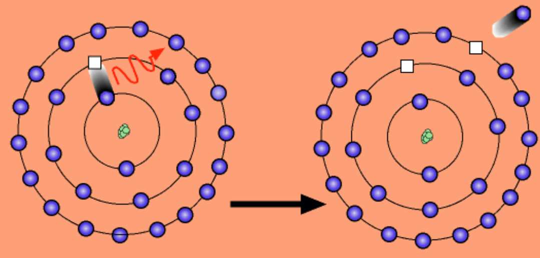
X-ray photon interacts with an atom and ejects one of orbital electrons.


Photoelectric Effect
hλ
hλ'
hλ″
Photoelectron
$T = h\nu - E_B - T_a$


Photoelectric Effect
Characteristic X-rays
hλ
hλ'
hλ″

Photoelectric Effect

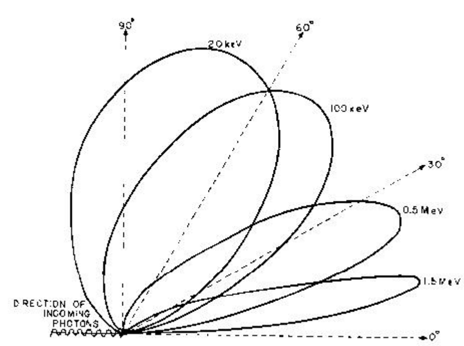
Auger Electrons



If the characteristice x-ray strikes an outer orbital electron the x-ray energy will be totally absorbed by the electron and the electron will be ejected from the atom. The electron will have energy characteristic of the element and is termed an Auger Electron.

Auger electrons are emitted as a competing process for characteristic x-rays

Photoelectrons



Interaction cross section

There are no simple formulas for photoelectric cross sections, this is because of complications due to the binding energy of the electron. Tables are based on experimental results and supplemented by theoretically assisted interpolations for other energies and absorbing media.

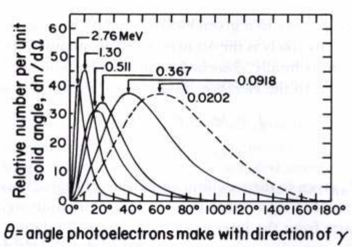


FIGURE 7.11. Directional distribution of photoelectrons per unit solid angle, for energies as labeled on the curves. The curve areas are not normalized to each other. [After Davisson and Evans (1952). Reproduced with permission of R. D. Evans and the American Physical Society.]

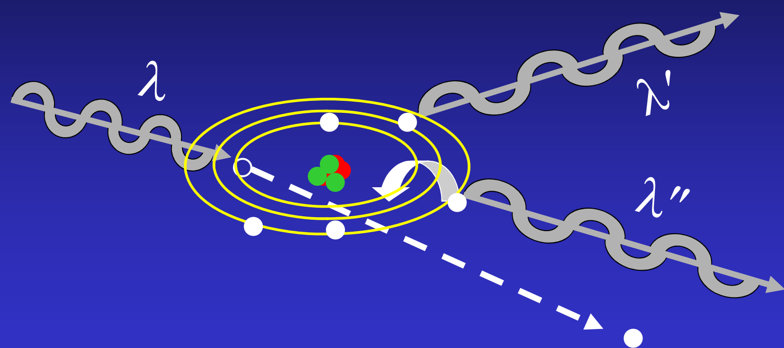
n = 4 at 0.1 MeV–> 4.6 at 3 MeV

$$_a\tau \cong k \frac{Z^n}{(h\nu)^m} \quad (\mathrm{cm^2/atom})$$

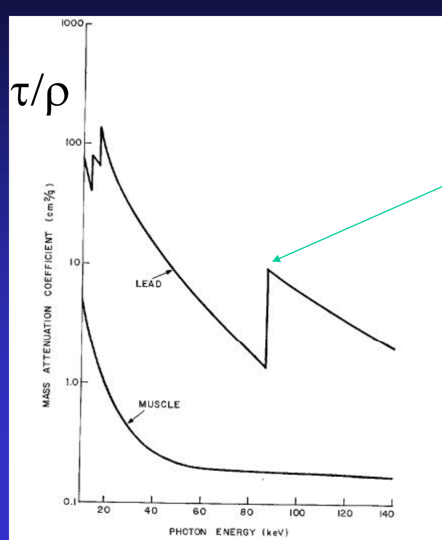
m = 3 at 0.1 MeV–> 1 at 5 MeV

$$\frac{\tau}{\rho} \propto \left(\frac{Z}{h\nu}\right)^3 \quad (\mathrm{cm^2/g})$$

Photoelectric Effect



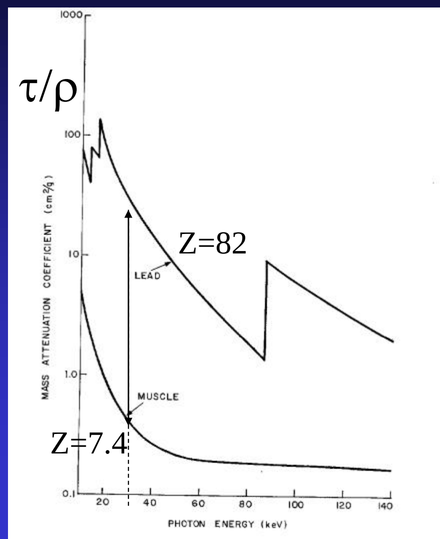
Photoelectric Effect



Characteristic

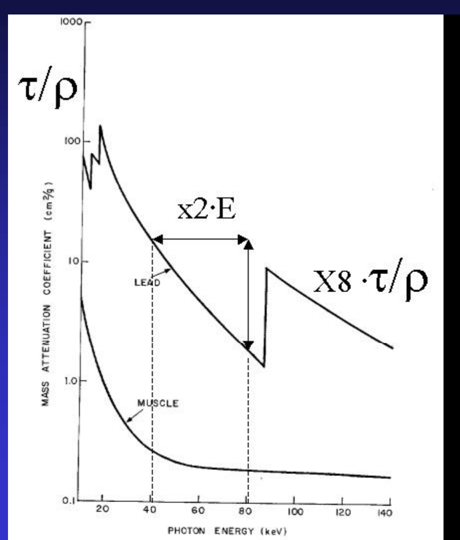
X-rays

Photoelectric Effect



$\tau/\rho \propto Z^3/E^3$

Photoelectric Effect



$\tau/\rho \propto Z^3/E^3$

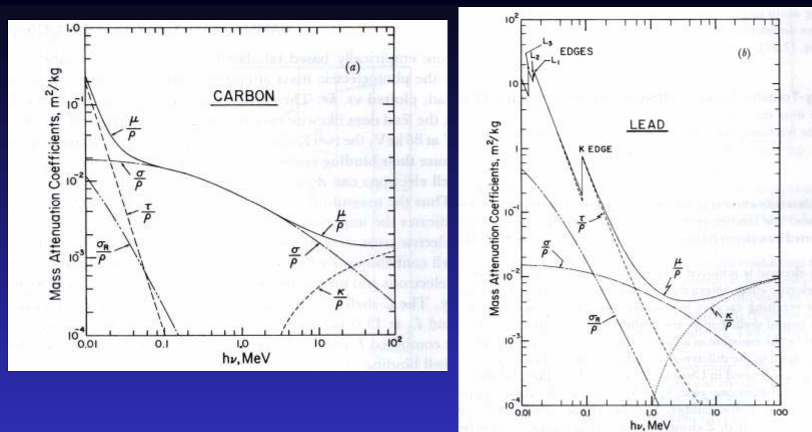


FIGURE 7.13. **Mass attenuation coefficients for carbon (a) and lead (b). τ/ρ indicates the contribution of the photoelectric effect, σ/ρ is that of the Compton effect, κ/ρ that of pair production, and σ_R/ρ that of Rayleigh (coherent) scattering. μ/ρ is their sum, which is closely approximated in Pb by the τ/ρ curve below $h\nu = 0.1$ MeV (From data of Hubbell, 1969).**

Energy-transfer cross section

$$\frac{T}{h\nu} = \frac{h\nu - E_b}{h\nu}$$

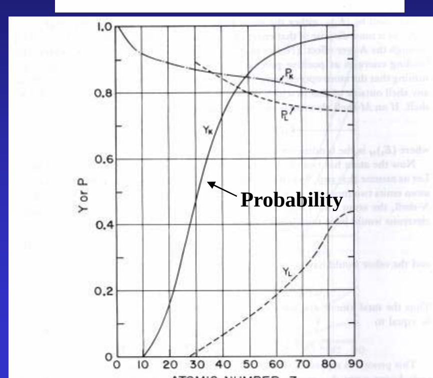


FIGURE 7.14. Fluorescence yield ($Y_{K,L}$) and fractional participation in the photoelectric effect ($P_{K,L}$) by K- and L-shell electrons (see text). P_K and P_L was calculated from tables of Hubbell (1969) and McMaster et al. (1969); Y_K from Lederer and Shirley (1979); and Y_L from Burhop (1952).

P_K is the fraction of all photoelectric interactions that occur in the K shell.

$P_K Y_K$ is the fraction of all photoelectric events in which a K fluorescence x-ray is emitted by the atom.

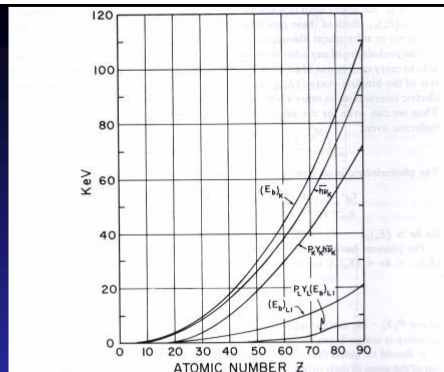


FIGURE 7.15. Electron binding energies $(E_b)_K$ in the K-shell and $(E_b)_{L1}$ in the $L1$-shell; weighted mean fluorescence x-ray energy $h\bar{\nu}_K$ in the K-shell; and the products $P_K Y_K \cdot h\bar{\nu}_K$ and $P_L Y_L (E_b)_{L1}$. The latter provides an upper-limit estimate of $P_L Y_L h\bar{\nu}_L$. Taken or derived from tables by Lederer and Shirley (1979).

$$\frac{\tau_{\text{tr}}}{\rho} = \frac{\tau}{\rho}\left[\frac{h\nu - P_K Y_K h\bar{\nu}_K - (1 - P_K) P_L Y_L h\bar{\nu}_L}{h\nu}\right]$$

$$\frac{\tau_{\text{tr}}}{\rho} = \frac{\tau}{\rho}\left[\frac{h\nu - P_L Y_L \cdot h\bar{\nu}_L}{h\nu}\right]$$